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APPLIED MECHANICS REVIEWS

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THE APPLICATION OF THE SHOCK TUBE TO THE STUDY OF HIGH TEMPERATURE PHENOMENA IN GASES

A. HERTZBERG

CORNELL AERONAUTICAL LABORATORY, INC.

INTRODUCTION

DURING the past ten years, the shock tube has come into widespread use as an experimental tool in gasdynamics research.* The U.S.A. appears to be the principal center of shock-tube research, with well-established groups at Aberdeen, Armour, Brown, Cal Tech, Cornell Aero. Lab., Cornell University, Lehigh, Los Alamos, Maryland, Michigan, NACA, NOL, and Princeton, and many new groups are now entering the field. Shock-tube research is also under way in Canada at UTIA, in Japan at the University of Tokyo, and in several laboratories in England and France.

The ability of the shock tube to simulate the temperatures and velocities of hypersonic flight has recently spurred its application to the study of high-temperature phenomena in gases. This paper will review the application of the shock tube to the study of high-temperature phenomena in gases and discuss some of the more important experiments and investigations that have recently been carried out. In particular, the application of the shock tube and its modified forms will be discussed in connection with the study of hypersonic flight, high-temperature chemical kinetics, and high-temperature gas physics.

HISTORICAL DEVELOPMENT

Early Research

The shock tube was originally devised by Vieille (48) in 1899 when it was used in connection with detonation experiments. However, it was not until 1937 that widespread interest was stimulated by the work of Payman and Shepherd (31) in England, who used the shock tube in studies of ignition. In this country the pioneering work of Bleakney (3,4) at Princeton was a major factor in establishing the shock tube as a useful tool for the study of gasdynamic phenomena. General articles surveying the field of shock-tube technique have been published by Bleakney (3), Glass and Patterson (13), and Penney and Pike (33), and a bibliography of much of the shock tube work prior to 1953 has been prepared by Elder (11). Texts by Courant and Friedrichs (9), Patterson (30), and Rudinger (39) deal with some of the theoretical and experimental aspects of shock-tube operation.

High-Temperature Studies

Many of the early investigators in gasdynamics observed that very high temperatures could be obtained behind strong shock waves. However, the early shock-tube studies did not exploit this capability. Actually, the early high-temperature experiments were carried out behind shock waves obtained by other techniques; for example, Zeldovich (51,52) fired high-speed bullets (up to 10,000 fps) into mercury vapor, and Muraour (26,28) used small explosive charges and studied emission spectra of heated gases. The technique of using explosives in the form of shaped charges and explosive lenses has been highly developed by the Los Alamos group (8,41), and has been extensively exploited in fundamental research in very strong shock phenomena. (The Los Alamos group has generated shock strengths of well over Mach 200 in air by explosive means.)

The extension of the conventional shock tube to high-temperature research was pioneered by Kantrowitz (18,34,35, 36) at Cornell University, who used the shock tube for many original studies of the nature of gases at high temperatures. At approximately the same time, independent work was being carried out by LaPorte's group (23,46) at Michigan, where an extensive study of the spectral radiation emitted by gases in shock tubes was started.

These successful programs aroused great interest in the further development of shock-tube techniques and, at the present time, the shock tube is widely used as a tool for high-temperature research in hypersonic aerodynamics, gas physics, and gas chemistry.

AERODYNAMIC TESTING

Conventional Shock Tube

The use of the conventional shock tube as a short duration wind tunnel was first explored by Geiger, Mautz, and Hollyer (12). In their experiments, they employed the flow that develops between the shock wave and the interface (i.e., the plane dividing the driver gas and the driven gas) to study short duration steady supersonic flows. This work was successful and the flow development about airfoils was intensively investigated. However, the usefulness of the shock tube in this form is limited by the maximum flow Mach number which can be obtained between the shock and the interface. For example, the maximum flow Mach number that can be obtained in this region in an ideal diatomic gas is 1.89, but, in air, the Mach number can be as high as Mach 3 because of real-gas effects (43). Consequently, the conventional shock tube is useful for aerodynamic investigations when it is not necessary to simulate the flight Mach number. It should be pointed out that since the velocity is continuous across the gas interface while the temperature drops sharply (40,49), the Mach

*The conventional shock tube consists of a duct separated by a diaphragm into high and low pressure regions. When the diaphragm is ruptured, a plane shock wave is generated which propagates into the low pressure region simultaneously accelerating the gas and raising its temperature.

number in the driver gas does not have the same limit. However, research workers who have attempted to exploit this region of flow have not been too successful, principally because of the very turbulent character of the flow (25). At Cornell Aeronautical Laboratory, it was felt that, if the Mach number limitation in the flow between the shock wave and the interface could be circumvented, the shock tube would prove to be a unique tool for hypersonic research.

At a flight Mach number of approximately 16, the equilibrium stagnation temperature at the nose of an insulated blunt body is approximately 6000 K in air at a density altitude of about 60,000 ft. At this temperature, the thermal energy of the air is sufficient to excite the higher rotational and vibrational states of the diatomic molecules, produce dissociation and ionization, and cause chemical reactions such as the formation of nitric oxide. New problems are consequently introduced since, in this temperature range, the air may be considered a chemically active fluid. For example, the convective heat transfer—the amount of heat transferred through the boundary layer to a body—may depend to a large extent upon the degree of dissociation and recombination inside and outside of the boundary layer. Thus, in this relatively unexplored temperature range, a primary need has been the development of laboratory facilities capable of creating the high temperatures in air which will be encountered in hypersonic flight.

Modified Shock Tube

Theoretical studies at Cornell Aeronautical Laboratory showed that, by terminating the end of the shock tube with a diverging nozzle, the low supersonic Mach number, high stagnation temperature flow behind the shock could be expanded to the desired test section Mach number and temperature. Indeed, calculations showed that a shock tube of this design could simulate hypersonic flight up to Mach 25. These theoretical calculations led to a research program to explore this approach. The experiments with this modified shock tube were successful, and it was found possible to exceed the flow Mach number limit of the straight shock tube (19). During this preliminary program, it was also observed that the flow in the nozzle was not established instantly, and the motion of the primary shock wave in passing through the nozzle set up a complex pattern of waves. It appeared that the direct application of the conventional shock tube with an expanding nozzle at flow Mach number above 7 would be difficult. Further examination of the starting phenomena led to the conclusion that, if the nozzle were highly evacuated prior to the shock tube experiment, the starting time could be minimized to less than 10% of the running time for practical hypersonic shock tunnels. This work on shock tunnel starting has been reported by Glick (14) and has been experimentally verified in shock tunnel tests (20).

Problems of Hypersonic Shock-Tunnel Application

While the shock tube equipped with a diverging nozzle is now coming into widespread use as a hypersonic shock-tunnel testing device (5,21,29,44,50), there are many problems involved in its use which have severely limited the full realization of its potential. One limitation, of course, is the extremely brief testing times that are available in devices of this type. For example, in the Cornell Aeronautical Laboratory shock tunnel, which has a low-pressure chamber approximately 40 ft long, the available testing time is less than one half millisecond at Mach 15, allowing for losses.

The brief testing time introduces the question of whether the flow over a model has sufficient time to become fully established. It is believed, based on experimental and theoretical grounds, that the wave pattern and boundary layer reach a steady state in appreciably less than one half millisecond for models approximately six inches long, since this time at Mach 15 corresponds to approximately fifteen model lengths of flow. However, it is not certain whether the wake flow is fully-developed in the short times available in the hypersonic shock tunnel, and investigations of the establishment of the wake flow are now in progress.

Also, the brief testing times make the problem of instrumentation extremely difficult. However, it has been found (7,47) that heat-transfer data can be obtained during a run by employing resistance thermometers of special design. It appears (47) that, if a thin metallic film about one-tenth micron thickness is bonded to the surface of a dielectric model, the temperature of the film corresponds to that of the surface with a lag which may be considered small as compared to the testing time. The heat-transfer rate at the surface during a run can be determined from the measured time history of the surface temperature. Techniques of instrumentation for the measurement of forces and pressures on the model in a hypersonic shock tunnel are also being investigated and, at the present time, a three-component balance system is being explored for shock-tunnel work (47).

Instrumentation for the measurement of pressure is being rapidly improved, and it is anticipated that in the near future pressure transducers will be available for use in shock-tunnel tests which should be sufficiently sensitive for the determination of static body pressures and have rise times on the order of microseconds. However, while the techniques of model instrumentation are being rapidly developed, little progress has been made in developing instrumentation for the measurement of the state of the flow in the test section; i.e., the absolute velocity of the flow, its density, and pressure under hypersonic shock-tunnel conditions.

In addition to the problems involved in shock-tube instrumentation, there are other problems which have been uncovered during the development of the hypersonic shock tunnel. Perhaps the most important of these at the present time is the problem of shock attenuation due to viscous forces. If reasonable testing times are to be obtained in shock tunnels, the length-to-diameter ratios of the low-pressure section must be extremely large in practical applications. For example, the Cornell Aeronautical Laboratory shock tunnel has a l/d in the low-pressure section of approximately 250, and any decrease in this value would be inconvenient. However, at these l/d's, the attenuation of strong shock waves becomes appreciable. As an indication of the seriousness of this shock attenuation, it was observed that an initial shock Mach number of approximately 18 attenuated to about 11 at the end of the tube. This attenuation results in a continuous variation in the density and velocity between the shock and interface.

These flow variations were not observed during the initial experimental work when schlieren and luminosity photographs were used as the principle method of nozzle instrumentation. However, when heat-transfer records were taken, it was observed that the heat-transfer rate at the nose of the model showed a variation during the run. Further examination showed that this heat-transfer variation was caused by continuous variation in the flow parameters at the nozzle caused by shock attenuation. Indeed, the heat-transfer gage has proven to be an extremely sensitive probe for shock-tube experiments.

At the present time, considerable attention is being devoted by various research groups to shock attenuation phenomena. Mirels (27), Trimpi and Cohen (45) have offered theories of attenuation in a shock tube, and several groups are now working on experimental verification and extension of these theories. In particular, a study of high Mach number shock attenuation is under way at the NACA under the direction of J. J. Jones. Jones has observed that the higher the speed of sound in the driver, the greater the shock attenuation.* (This result has been confirmed in tests at C.A.L.) Since high driver acoustical velocities are required for driving strong shocks, the application of the shock tunnel at high Mach numbers in slender tubes may be extremely difficult.

It should be pointed out that shock tubes of more sophisti-

*In private communication with the author

ated design may alleviate the attenuation problem. For example, at Cornell Aeronautical Laboratory, a reflected shock technique of operation is now being extensively studied which promises an effective increase of testing time per foot of shock-tube length of nearly 25 times that of the conventional shock tube (21,22). This technique involves the insertion of a converging-diverging nozzle at the end of the low-pressure chamber and the proper selection of the driver gas. Preliminary experiments at C.A.L. have demonstrated that this technique can be carried out successfully and a program of exploring this modification as a method for relieving the attenuation problem in the shock tube is now under way.

SHOCK-TUBE STUDIES IN CHEMISTRY

The application of the shock tube to the study of high-speed chemical reactions goes back again to the pioneering work of Payman and Shepherd (31,32). Since then, the use of the shock tube for the study of high-speed chemical reactions has become widespread. While the ability of the shock tube to heat gases to high temperatures is important in chemical studies, it is of only secondary importance when compared to the precision with which the state of the gas can be monitored. Indeed, if the shock velocity is measured, the enthalpy of the gas behind the shock wave may be accurately determined. The rate of heating is small and heat losses due to radiation and conduction may be safely neglected. Further, the shock wave processes the gas homogeneously and uniformly so that when the shock tube is used as a chemical reactor, the spatial variation of properties in the reacting mixture is small. The presence of gradients facilitates studies using spectroscopy, schlieren and interferometry techniques, as well as gas sampling methods.

The shock tube has also been used in the study of detonation and combustion phenomena in gaseous mixtures, but this work is beyond the scope of this paper.

Applications to Chemical Kinetics

An early application of the shock tube to chemical kinetics was developed by Davidson (6) and his associates at Cal Tech in their studies of dissociation reactions. In these experiments, gases, such as N_2O_4 which is opaque, were dissociated by weak shock waves and the kinetics of the reaction was followed by optically observing the change in gas opacity with time. This technique has been extended by Hornig (16,17) and his associates at Brown University, where emission and absorption spectroscopy has been employed to study the reacting mixture immediately behind the shock. The appearance of various molecular and atomic species was detected and provided data on the chemical kinetics. At the present time, the combination of the shock tube and spectroscopy is receiving considerable attention from other research groups, and this technique is expected to find a useful application in the field of chemical kinetics.

However, shock-tube spectroscopy is not without its problems. For example, the reaction velocities may be high and the radiation may be of low intensity. It is extremely difficult to obtain useful time-resolved spectra under these conditions. Also, shock-tube spectra are often confused by the presence of impurities. It must be pointed out that, in the application of the shock tube to high-temperature research, impurities are extremely difficult to eliminate and great care must be taken in this phase of the work. The difficulty is caused by the fact that the shock-tube interior must be frequently exposed to the atmosphere and to the severe conditions imposed by the driver which tend to leave impurities in the shock tube.

Another shock-tube modification that obviates some of the problems discussed above is being developed at Cornell Aeronautical Laboratory. In this technique, a conventional shock tube is modified so that the reactant gas sample can be processed by a single, controlled high-temperature pulse. The geometry of the shock tube is so arranged that the reactant gas is first compressed and heated by the primary and reflected shock waves. The reactant gas is then allowed to remain at the reaction temperature and pressure for a specified period of time during which the chemical reaction proceeds. The reactant gas is then suddenly cooled by an expansion wave. If the cooling rate is very high, the reaction rates can be reduced so rapidly that the reaction is "frozen" and the products that exist at the first instant of cooling remain after the run and can be analyzed by examining samples of the gas left in the shock tube. It should be pointed out, however, that this technique is limited to the study of those reactions and products which can be frozen. For example, free radicals which are created at the high-temperature conditions would react very rapidly even after cooling so that their presence could not easily be detected.

A description of the operation of the Cornell Aeronautical Laboratory chemical shock tube may be found in reference 15. This shock tube has been used to study in detail the reaction $N_2 + O_2 \rightleftarrows 2NO$. This reaction, it is interesting to note, is also of importance in hypersonic flight since, at high stagnation temperatures, a considerable amount of nitric oxide may be formed in the vicinity of the body and may remain frozen out after the passage of the body. In the chemical shock tube, it was possible to study the kinetics of nitric oxide formation in the temperature range from 2000K–3000K over a wide range of operating pressures. From the observed data, rate constants for the forward and reverse reactions were calculated assuming a bimolecular reaction, which has proven adequate at lower temperatures. It was observed that in the temperature range of 2000K–3000K, different reaction rate constants were obtained at a given temperature for different oxygen concentrations. These results indicated that the reaction did not agree with the bimolecular hypothesis. A literature search showed that the chemical shock-tube data agreed very closely with results that Zeldovich obtained in the same temperature range using a combustion bomb (53). Zeldovich proposed a chain mechanism to explain his data. This chain mechanism implies that when the data are reduced assuming the "bimolecular" mechanism, the reaction-rate constants vary inversely with the square root of the oxygen concentration, which is in essential agreement with results of the C.A.L. investigation.

In the Zeldovich chain mechanism hypothesis, it is assumed that oxygen dissociation reaches equilibrium in a time small compared with that for the formation of nitric oxide. Theoretical calculations (1), however, indicate that the relaxation time for oxygen dissociation is far longer than the time for the formation of nitric oxide as determined by this experimental investigation. Thus, the experiments performed in the chemical shock tube suggest that the model employed in these theoretical calculations be re-examined. Additional experimental work, which is described in the next section, indicates that dissociation relaxation times in oxygen are indeed small, and that the assumption of the Zeldovich chain is correct.

SHOCK-TUBE STUDIES IN PHYSICS

The high-temperature potential of the shock tube has given the experimental physicist a simple and convenient tool for appreciably extending the range of study of high-temperature phenomena in gases. It should also be pointed out that, although it is possible to heat gases to fairly high temperatures by other means, the ability of the shock tube to produce a homogeneous gas sample in a well-defined thermodynamic state is noteworthy. These advantages, which are peculiar to shock-tube techniques, are presently being exploited at many laboratories both in this country and abroad.

The earliest radiation studies in shock tubes were carried out at Cornell University by Kantrowitz (35) and his group, who studied the radiation from argon. These workers were able to observe a second-order Stark effect in argon and study the character of the continuum radiation. Similar studies in argon, krypton, neon, and xenon were carried out at the University of Michigan by LaPorte (46) and his associates. Other

shock-tube work by Kantrowitz's group has included conductivity studies (24) and magneto-hydrodynamics experiments.

Shock-tube work has been carried out at Los Alamos by Schreffler and his associates, who have used explosive-driven shock tubes capable of generating strong shock waves. Since the work at Los Alamos has included experiments at shock strengths on order of magnitude above those at other laboratories, several singular phenomena have been observed. For example, the radiation intensity from argon heated by strong shocks is very great and preheats the gas in front of the advancing shock wave, expecially near the boundaries. The shock front is therefore distorted by the nonuniform radiation heating of the gas in front of the advancing shock wave (41). Another important related study carried out at Los Alamos (8) was the determination of the dissociation energy of nitrogen by precisely measuring the space-time history of the shock and gas interface.

Vibrational relaxation studies have been carried out at Princeton by Bleakney (2) and his colleagues, and at Naval Ordnance Laboratory (42), using a spark interferometer technique to examine the density gradients behind shock waves moving into oxygen, nitrogen, air, and CO_2. Similar experiments of dissociation relaxation are now in progress. Relaxation studies have also been carried out at the University of Maryland by Resler (37) and his group, who have observed several anamolous relaxation phenomena.

It should be pointed out that the same sort of advantages and disadvantages that apply to aerodynamic and chemical investigations in the shock tube apply to physics studies. Short duration of phenomena, boundary-layer effects, impurities in the gases have challenged the shock-tube experimentalist.

Physics studies at C.A.L. have generally been restricted to studies of the high temperature properties of air and its chief components, oxygen and nitrogen. In addition to the experimental shock-tube program, a theoretical program has been underway for several years to calculate the thermodynamic and transport properties of air, oxygen, and nitrogen up to temperatures of 10,000K over a wide range of densities.

The first experimental work at C.A.L. was a study of the physics of the flow behind strong shock waves using schlieren optical techniques. The schlieren pictures clearly showed two characteristics of high temperature air. First, at the high temperatures produced behind strong shocks, air does not behave like a gas with a constant heat capacity, but rather, vibration and dissociation become important and contribute to the heat capacity. The increase in the heat capacity of air due to vibration and dissociation appreciably affected the observed flow Mach number. Second, pictures taken with a vertical schlieren knife edge showed a relaxation zone immediately behind the shock, similar to those observed in Bleakney's relaxation studies.

These first studies showed important areas of investigation that could be carried out conveniently in the shock tube. Additional experiments have been under way for some time in which the schlieren technique for studying relaxation has been applied to oxygen and nitrogen. Preliminary results have been obtained in oxygen which show that, in temperature and density ranges where vibrational relaxation is able to be recorded with this technique, there is good agreement with the vibrational data of Blackman (2). However, at higher temperatures, a new relaxation zone has been observed by Glick and Wurster which tentatively has been identified as dissociation relaxation. The measured times of dissociation relaxation for oxygen are, however, orders of magnitude smaller than those predicted in reference (1). The experiments in oxygen are now being repeated with high gas purity. If the preliminary results are verified in these studies, the theoretical mechanism for oxygen dissociation will require further investigation (1,38).

Another study that is currently being pursued at C.A.L. is the study of radiation from air up to temperatures of 8000K. Emission spectra of air at temperatures from 6000K to 8000K have been taken. These spectrograms show several molecular bands, and the resolving power is sufficient to resolve many of the rotational lines. The spectrograms have not been fully analyzed, but band systems of NO and OH were clearly visible from shock-heated room air. It is interesting to note that the CN band, which has plagued shock-tube spectroscopy in the past, did *not* appear when a sufficient purity was achieved. In addition to the emission studies, absorption studies of air are planned for the future.

CONCLUSIONS

The shock tube has developed, during the past ten years, into a powerful tool for the study of high-temperature phenomena in gases. The experimental technique is rapidly advancing and it is expected that the shock tube will find increasing applications in the field of hypersonic flight, high-temperature chemical kinetics, and high-temperature gas physics.

1. *Hypersonic Flow Research*

The conventional shock tube is capable of producing air temperatures which are high enough to cover the range of hypersonic flight up to a Mach number of 25. However, in order to simulate both the high temperatures of hypersonic flight and the high flow Mach numbers, the conventional shock tube must be modified. The most successful modification, at the present time, has been the addition of a diverging nozzle to the downstream end of the low-pressure chamber to increase the flow Mach number. While this modification has proven successful for many flow research problems, the extremely brief testing times possible at high Mach numbers (approximately one-half millisecond) introduce the problem of flow establishment and complicate the problem of instrumentation. Also, the quest for longer testing times requires the use of relatively long shock tubes and introduces the problem of severe shock attenuation. New modifications of the shock tube are now being proposed and explored which promise to alleviate this attenuation problem and extend the range of shock-tube operation.

2. *Application of the Shock Tube to Chemistry*

Shock-tube spectroscopy has been successfully applied to the study of high-speed chemical kinetics.

A modified form of the shock tube has been developed which generates a single, high-temperature pulse. This technique brings the gas to the reaction temperature suddenly, maintains the reaction conditions for a fixed period of time, and then rapidly cools the reacting gas. Reactions that can be frozen by the sudden quenching available in this shock-tube modification can be studied in detail. In particular, the reaction of $N_2 + O_2 \rightleftarrows 2NO$ has been studied, and the data have been correlated with a chain mechanism.

3. *Application of the Shock Tube to Physics*

The high-energy shock tube brings the physics of high-temperature gas phenomena a new research tool with which to study gases under known conditions at high temperatures.

The shock tube in this form has been, and is now being, actively applied to the measurement of the equation of state of gases, the study of atomic and molecular spectra, the measurement of electrical conductivity, and the kinetics of the establishment of thermal equilibrium. In particular, a great deal of attention is now being devoted to the measurement of relaxation times to dissociation for oxygen and nitrogen. Preliminary experiments have shown extremely brief relaxation times for dissociation in oxygen.

REFERENCES

1 Bethe, H. A., and Teller, E., Deviations from thermal equilibrium in shock waves, *BRL Rep.* X117.

2 Blackman, V. H., Vibrational relaxation in oxygen and nitrogen, *J. fluid Mech.* 1, p. 61, 1956.

3 Bleakney, W., and Taub, A. H., Interaction of shock waves, *Rev. mod. Phys.* **21**, p. 584, 1949.
4 Bleakney, W., Weimer, D. K., and Fletcher, C. H., The shock tube: A facility for investigations in fluid dynamics, *Rev. sci. Instr.* 20, p. 807, 1949.
5 Busing, J. R., Reynolds numbers in the hypersonic shock tunnel, English Electric Co., Ltd., Rep. LA.t.074, Feb. 14, 1956.
6 Carrington, T., and Davidson, N., Photoelectric observation of the rate of dissociation of N_2O_4 by a shock wave, *J. Chem. Phys.* 19, p. 1313, 1951.
7 Chabai, A. J., and Emrich, R. J., Measurements of wall temperature and heat flow in the shock tube, *J. appl. Phys.* **26**, 6, 779–780, June 1956; AMR **8**, Rev. 3885.
8 Christian, R. H., Duff, R. E., and Yarger, F. L., Equation of state of gases by shock wave measurements, 2-the dissociation energy of nitrogen, *J. Chem. Phys.* **23**, p. 2045, 1955.
9 Courant, R., and Friedrichs, K. O., Supersonic flow and shock waves, New York, Interscience Publishers, Inc., 1948; AMR 1, Rev. 981.
10 Cowan, G. R., and Hornig, D. F., The experimental determination of the thickness of a shock front in a gas, *J. Chem. Phys.* **18**, p. 1008, 1950.
11 Elder, F. K., Shock wave bibliography of periodical literature, GEL JHU TG 75–3, Mar. 1953.
12 Geiger, F. W., Mautz, C. W., and Hollyer, R. N., The shock tube as an instrument for the investigation of transonic and supersonic flow patterns, *Univ. Mich., Eng. Res. Inst.*, Project M720–4, June 1949.
13 Glass, I. I., and Patterson, G. N., A theoretical and experimental study of shock tube flows, *J. aero. Sci.* **22**, 2, 73–100, Feb. 1955; AMR **8**, Rev. 2486.
14 Glick, H. S., Hertzberg, A., and Smith, W. E., Flow phenomena in starting a hypersonic shock tunnel, *Cornell Aero. Lab. Rep.* AD–789–A–3, AEDC–TN–16, Mar. 1955.
15 Glick, H. S., Squire, W., and Hertzberg, A., A new shock tube technique for the study of high temperature gas phase reactions, Fifth Symposium (International) on Combustion, New York, Reinhold Publishing Corp., 1955. pp. 393–402.
16 Greene, E. F., Chemical reactions in strong shock waves, *J. Amer. chem Soc.* **76**, p. 2127, 1954.
17 Greene, E. F., Cowan, G. R., and Hornig, D. F., The thickness of shock fronts in argon and nitrogen and rotational heat capacity lags, *J. Chem. Phys.* **19**, p. 427, 1951.
18 Hertzberg, A., and Kantrowitz, A., Studies with an aerodynamically instrumented shock tube, *J. appl. Phys.* **21**, p. 874, 1950.
19 Hertzberg, A., A shock tube method of generating hypersonic flows, *J. aero. Sci.* **18**, p. 803, 1951.
20 Hertzberg, A., The application of the shock tube to the study of the problems of hypersonic flight, *Jet Propulsion* **26**, p. 549, 1956.
21 Hertzberg, A., Smith, W. E., Glick, H. S., and Squire, W., Modifications of the shock tube for the generation of hypersonic flow, *Cornell Aero. Lab. Rep.* AD–789–A–2, AEDC–TN–55–15, Mar. 1955.
22 Hertzberg, A., and Logan, J. G., The application of the shock tube to the study of the aerothermal problems of high-speed flight, Proceedings of Conference on Chemical Aeronomy, June 1956 (To be published).
23 Hollyer, R. N., Hunting, A. C., LaPorte, O., Schwarcz, E. N., and Turner, E. P., Luminous effects in the shock tube, *Phys. Rev.* 85, p. 911, 1952.
24 Lin, S. C., Resler, E. L., and Kantrowitz, A., Electrical conductivity of highly ionized argon produced by shock waves, *J. appl. Phys.* **26**, p. 95, 1955.
25 Lobb, R. K., A study of supersonic flows in a shock tube, *UTIA Rep.* no. 8, 43 pp. + 58 figs., May 1950; AMR **5**, Rev. 2677.
26 Michel-Levy, A., Muraour, H., and Vassy, E., Distribution of energy in the light emitted on the encounter of shock waves, *Rev. Opt.* **20**, p. 149, 1941.
27 Mirels, H., Attenuation in a shock tube due to unsteady boundary layer action, *NACA TN* 3278, Aug. 1956.
28 Muraour, H., Notes on the continuous spectra obtained by collision between shock waves in different gases, *Cahiers Phys.* **29**, p. 54, 1947.
29 Parks, E. K., Supersonic flow in a shock tube of divergent cross-section, *UTIA Rep.* no. 18, 31 pp. + 51 figs., May 1952; AMR **5**, Rev. 3492.
30 Patterson, G. N., Molecular flow of gases, New York, John Wiley & Sons, Inc., 1956.
31 Payman, W., and Shepherd, W. C. F., Explosion waves and shock waves, Part IV-Quasi-detonation in mixtures of methane and air, *Proc. roy. Soc. Lond. (A)* **158**, p. 348, 1937.
32 Payman, W., and Shepherd, W. C. F., Explosion waves and shock waves, Part VI-The disturbance produced by bursting diaphragms with compressed air, *Proc. roy. Soc. Lond. (A)* **186**, p. 293, 1946.
33 Penney, W. G., and Pike, H. H. M., Shock waves and propagation of finite pulses in fluids, Reports of Phys. Soc. on Prog. Phys. **13**, p. 46, 1950.
34 Perry, R. W., and Kantrowitz, A., The production and stability of converging shock waves, *J. appl. Phys.* **22**, 7, 878–886, July 1951; AMR **5**, Rev. 176.
35 Petschek, H. E., Rose, P. H., Glick, H. S., Kane, A., and Kantrowitz, A., Spectroscopic studies of highly ionized argon produced by shock waves, *J. appl. Phys.* **26**, p. 83, 1955.
36 Resler, E. L., Lin, Shao-Chi, and Kantrowitz, A., The production of high temperature gases in shock tubes, *J. appl. Phys.* **23**, 12, 1390–1399, Dec. 1952; AMR **6**, Rev. 2038.
37 Resler, E. L., and Scheibe, M., Instrument to study relaxation rates behind shock waves, *J. acoust. Soc. Amer.* **27**, p. 932, 1955.
38 Rice, O. K., Reply to Careri's note on the rate of recombination of free atoms, *J. Chem. Phys.* **21**, p. 750, 1953.
39 Rudinger, G., Wave diagrams for nonsteady flow in ducts, Toronto, New York, London, D. Van Nostrand, Inc., 1955, xi + 278 pp.; AMR **8**, Rev. 3110.
40 Rudinger, G., Use of the shock tube as an intermittent supersonic wind tunnel, *Phys. Rev.* **75**, p. 1948, 1949.
41 Schreffler, R. G., and Christian, R. H., Boundary disturbance in high explosive shock tubes, *J. appl. Phys.* **25**, p. 324, 1954.
42 Smiley, E. F., Winkler, E. H., and Slawsky, Z. I., Measurement of the vibrational relaxation effect in CO_2 by means of shock tube interferograms, *J. Chem. Phys.* **20**, p. 924, 1952.
43 Squire, W., Hertzberg, A., and Smith, W. E., Real gas effects in a hypersonic shock tunnel, *Cornell Aero. Lab. Rep.* Ad–789–A–1, AEDC–TN–55–14, Mar. 1955.
44 Tamaki, F. A., A divergent shock tube for obtaining supersonic flow, *J. phys. Soc. Japan* **11**, p. 434, 1956.
45 Trimpi, R. L., and Cohen, N. B., A theory for predicting the flow of real gases in shock tubes with experimental verification, *NACA TN* 3375, Mar. 1955; AMR **8**, Rev. 3508.
46 Turner E. B., The production of very high temperatures in the shock tube with an application to the study of spectral line broadening, *Univ. Mich. Rep.* AFOSR TN–56–150, May 1956.
47 Vidal, R., Model instrumentation techniques for heat transfer and force measurements in a hypersonic shock tunnel, *Cornell Aero. Lab. Rep.* AD–917–A–1, WADC–TN–56–315, Feb. 1956.
48 Vieille, P., Sur les discontinuites produites par brusque des gas comprimes, *C. R. Acad. Sci. Paris* 129, 1899.
49 Wegener, P., and Lundquist, G., Condensation of water vapor in the shock tube below 150° K, *J. appl. Phys.* **22**, p. 233, 1951.
50 Yoler, Y. A., Hypersonic shock tube, GALCIT Memo no. 18, July 1954.
51 Zeldovich, Ya. B., and Leipunsky, O., Obtaining of extremely high temperatures, *J. Phys., URSS* **7**, p. 245, 1943.
52 Zeldovich, Ya. B., and Shlyapintokh, I. Ya., Ignition of explosive gas mixtures in shock waves, *Doklady Akad Nauk SSSR* **65**, p. 871, 1949.
53 Zeldovich, Ya. B., The oxidation of nitrogen in combustion and explosions, *Acta Physicochimica, URSS* **21**, p. 577, 1946.

"Letters to the Editor" and "Books Received for Review" appear after the reviews

Theoretical and Experimental Methods

(See also Revs. 3828, 3831, 3835, 3836, 3837, 3838, 3841, 3842, 3848, 3849, 3884, 3885, 3901, 3904, 3917, 3928, 3929, 3970, 3982, 3984, 3985, 4019, 4039, 4064, 4065, 4066, 4069, 4090, 4125, 4147)

Book—3806. EL'sgol'ts, L. E., Qualitative methods in mathematical analysis [Kachestvenniye metodi v matematicheskom analize], Moscow, Gos. Izdat. Tekh.-Teor. Lit., 1955, 300 pp. 10r. 15k.

Subject matter concerns various methods for recognizing special properties of a solution without solving the associated mathematical problem. Brief selections from several chapters will give a more detailed idea on the content and purpose of the book.

Chapter 1 deals (66 pp.) with qualitative methods in problems on extrema of functions and functionals. Here the most substantial role belongs to the abstract space in which the functional or function in question are defined. The following section (13 pp.) studies problems from the theory of functions, both of one and of many complex variables. Third main part shows, by various examples, the powerful possibilities of the well-known Banach's principle of fixed points. The study of these three chapters presupposes adequate familiarity with topology.

Fourth section deals with typical qualitative methods from the field of differential equations (dynamical systems, stability and periodicity of solutions—altogether 86 pp.) Especially valuable is the material of the following chapter (73 pp.) devoted to hysterodifferential and related equations. These topics are of eminent importance in modern engineering; e.g., for automatic regulation. Of special interest is the last section (16 pp.) on variational problems with retarding argument. Work concludes with a rich index of literature (in all, 339 title sources).

Paper and print are good; the clearness of presentation and readability of the book are considerably heightened by 52 instructive figures. This is a very valuable work, doubtless presenting much of the material for the first time. We recommend it warmly not only to every mathematician but especially also to calculating engineers and physicists. V. Vodička, Czechoslovakia

Book—3807. Sneddon, I. N., Special functions of mathematical physics and chemistry, New York, Interscience Publishers, Inc., 1956, viii + 164 pp. $1.75.

Author presents the special functions arising in the solution of some common differential equations. Book covers hypergeometric, Legendre, Bessel, Hermite, and Laguerre functions, and concludes with a note on the Dirac delta function. A typical chapter contains a definition of the function in series form, integral expressions, a description of the most important properties, a statement of the equations to which the function is a solution, an example of its use in dealing with some physical problem, and an appendix of problems for the reader.

The treatment is at the level of a first-year graduate course. The presentation is concise and readable.

This surprisingly slim, inexpensive, and functional book makes one regret that the economics of the publishing industry in this country almost eliminate the pocket-size textbook.

M. C. Junger, USA

3808. Zbornik, T., Solution of certain homogeneous linear second-order equations (in German), *ZAMP* **7**, 1, 64–74, Jan. 1956.

Complicated differential equations often transform to solvable forms by a suitable transformation of variables. Author transforms the linear equation $Y'' + GY' + HY = 0$ by putting $y(x) = v(u)$, $u = \int \exp(-\int G dx) dx$ into $(du/dx)^2 \cdot (d^2v/d^2u) + Hv = 0$. There are many cases where this latter equation can be solved by elementary methods, and from the result $v = v(u)$ follows immediately the desired solution $y(x)$.

Author treats 5 types of equations solvable by the above procedure. He gives altogether 66 examples, many of which cannot be found even in such voluminous sources as the well-known handbook by Kamke. Treated are, for instance, several equations leading to cylindrical functions. The general form of equations tractable by the method under consideration is also given.

Paper is an interesting contribution to the subject. Numerous equations solved in the paper can be of importance in technical problems. V. Vodička, Czechoslovakia

3809. Olver, F. W. J., Errors in asymptotic solutions of linear ordinary differential equations, *Quart. appl. Math.* **14**, 2, 218–219 (Notes), July 1956.

3810. Poritsky, H., Expression of wave functions over a half space in terms of their boundary values, *Quart. appl. Math.* **14**, 2, 185–197, July 1956.

3811. Germain, P., An expression for Green's function for a particular Tricomi problem, *Quart. appl. Math.* **14**, 2, 113–124, July 1956.

Green's function for the Tricomi equation (introduced for the first time in AMR **6**, Rev. 1666), which allows the solution of the Tricomi problem to be written as a linear functional of the boundary data, is deduced in a new way. The main points of this method are: (1) each "normal" Tricomi problem can be brought in a problem with defined universal boundary by applying a certain three-parametric transformation related to the Poincare geometry; (2) thereafter, new independent variables can be introduced into the Tricomi equation in such a way that Green's function of the Tricomi problem to the special boundary reached in (1) is transformed into Green's function of a strip in a mixed domain for a certain new differential equation; (3) this last mentioned Green's function can be constructed with a method analogous to that applied in an earlier paper of the author's [*Bull. Soc. math. France*, 1953].

Compared to the expression of Green's function given in AMR **6**, Rev. 1666, the new form is shown to be more satisfactory from the computational point of view. This is of importance for applications to the approximate calculation of subsonic and transonic flows. H. Behrbohm, Sweden

3812. Lax, A., On Cauchy's problem for partial differential equations with multiple characteristics, *Comm. pure appl. Math.* **9**, 2, 135–169, May 1956.

This clearly written paper is concerned with algebraic criteria for the proper solvability of Cauchy's initial-value problem for linear partial differential equations with real coefficients for a function of two independent variables. Let $L[u] = 0$ be an equation of any order m; in case of constant coefficients, let $a_{i,j}$ be the coefficient of $\partial^{i+j} u/\partial x^i \partial t^j$; and associate the polynomial

$$p_k(z) = a_{0,k} z^k + a_{1,k-1} z^{k-1} + \ldots + a_{k,0}$$

($k = 1, 2, \ldots m$) with the homogeneous operator of order k in $L[u]$. Then, given any noncharacteristic initial curve, necessary and sufficient conditions for solvability of Cauchy's problem are

(1) the roots of $p_m(z) = 0$ are real,

(2) if λ is an r-fold root of $p_m(z) = 0$, i.e., $(z - \lambda)^r$ is a factor of $p_m(z)$ while $(z - \lambda)^{r+1}$ is not, then $(z - \lambda)^{r-s}$ is a factor of $p_{m-s}(z)$ for $s = 1, 2, \ldots r-1$.

The main results in case of variable coefficients are two equivalent criteria for solvability. R. Albrecht, Germany

3813. Wunderlich, W., Formulas and computational aids for the development of the surface of a second-order cone (in German), *Öst. Ing.-Arch.* **10**, 1, 107–114, 1956.

By a cone of second order is meant a cone with an ellipse as basic area, while the projection of the apex on this area is at the center of the ellipse. For such cones a formula is derived expressing the angle of the developed round surface in terms of the maximum and minimum opening angles of the cone. Since this formula contains complicated elliptic integrals, results have also been presented in a table and a diagram. An approximate formula with corresponding nomogram is also given.

A. I. van de Vooren, Holland

Book—3814. Hartree, D. R., Numerical analysis, corr. 1st ed., New York, Oxford University Press; Oxford, The Clarendon Press, 1955, xiv + 287 pp. $4.80.

Author states that the book is introductory in that no previous knowledge of systematic numerical work is assumed, but is not elementary in the sense of using only arithmetic. A knowledge of calculus, including series and partial derivatives, differential equations, and simple properties of matrixes, is assumed.

Author emphasizes the important factors of proper formulation of the problem for numerical work, errors due to dropping terms in series and due to rounding off, and checking for mistakes. He points out that, in some cases, it may be easier to make a numerical integration of a differential equation directly than to calculate from the formula obtained by solution of the equation.

Chapters in the book are the tools of numerical work and how to use item: evaluation of formulas, finite differences, interpolation, integration (quadrature) and differentiation, integration of ordinary differential equations, simultaneous linear algebraic equations and matrixes, nonlinear algebraic equations, functions of two or more variables, miscellaneous processes, and organization of calculations for an automatic machine.

Book is primarily for numerical work using desk machines, with the last chapter on programming for automatic machines. On the whole, book appears to be well arranged and to present the fundamentals for numerical analysis needed in engineering work.

B. E. Gatewood, USA

3815. **Flemming, D. P., Iterative procedure for evaluating a transient response through its power series,** *Math. Tables Aids Comput.* **10,** 54, 73–81, Apr. 1956.

Coefficients in power series in the time are obtained one after the other from coefficients in the numerator and denominator polynomials in the transform. Overestimate of number of terms needed for given accuracy is also provided. Method is very suitable for machine computations. C. Ablow, USA

3816. **Chisnall, G. A., A modified Chebyshev-Everett interpolation formula,** *Math. Tables Aids Comput.* **10,** 54, 66–73, Apr. 1956.

Consider the closed interval (-1,1). It is known that of all polynomials with leading coefficient unity, the Chebyshev polynomials of the first kind have least deviation from zero. Rearrangement of a power series as a series of Chebyshev polynomials gives an economic series in that maximum accuracy is attained with a small number of terms. Purpose of note is to rearrange the Everett interpolation formula in a series of Chebyshev polynomials. This technique has been called optimum interpolation. See Appendix II of Kopal's volume on "Numerical analysis" [MR **9,** Rev 2796]. Y. L. Luke, USA

3817. **Henrici, P., A subroutine for computations with rational numbers,** *J. Assn. Comp. Machy.* **3,** 1, 6–9, Jan. 1956.

Paper contrasts the perfect accuracy attainable in numerical computation using integers with the troublesome errors which arise from introducing truncated binary or decimal fractions into calculations. Hence, author suggests that when computations deal exclusively with rational numbers, these numbers should be represented as ordered pairs of integers rather than as truncated binary or decimal fractions. Computer routines for arithmetic with rational numbers represented as ordered pairs of integers are described. A knowledge of a particular computer (SEAC) is helpful but not essential. Unfortunately, the ideas presented will have little application to the usual engineering computation.

M. L. Stein, USA

3818. **Bauer, F. L., Abbreviated iteration procedure for algebraic eigenvalue problems, particularly for the determination of zeros of a polynomial** (in German), *ZAMP* **7,** 2, 17–32, 1956.

Lagrange's version of Bernoullian iteration method for solving the equation $P(x) = x^n + a_1 x^{n-1} + \cdots + a_n = 0$ consists of successively determining $Q^{(i)}(x) = y_1^{(i)} x^{n-1} + y_2^{(i)} x^{n-2} + \cdots + y_n^{(i)}$, the remainder of $x^i Q(x)$ divided by $P(x)$, $(i = 1,2,\cdots)$, where $Q(x)$ is a starting polynomial. Actual computation follows the formula $Q^{(i+1)}(x) = xQ^{(i)}(x) - y_1^{(i)} P(x)$. If $P(x)$ has n roots ξ_1, $\xi_2, \cdots, \xi_n$ such that $|\xi_1| > |\xi_2| > \cdots > |\xi_n| > 0$, then, starting with $Q(x) = 1$, $Q^{(i)}(x)/y_1^{(i)}$ converges to $P_1(x) = P(x)/(x - \xi_1)$.

Next, starting with $Q(x) = 1 - P_1(x)/P_1(\xi_1)$, $Q^{(i)}(x)/y_1^{(i)}$ converges to $P_2(x) = P(x)/(x - \xi_2)$, and so on.

Author proposes following abbreviation of the above process:

$$Q^{(i+1)}(x) = y_1^{(il)} Q^{(i+n-1)}(x) + y_2^{(i)} Q^{(i+n-2)}(x) + \cdots + y_n^{(i)} Q^{(i)}(x),$$

where the polynomials $Q^{(i+1)}(x), \cdots, Q^{(i+n-1)}(x)$ are to be determined by the ordinary Bernoullian iteration. It secures second-order convergence. Divisions can be avoided. Rounding error is not carried. Each iteration run is self-correcting. Checking is easy. Author says it has been tried out in extensive computations.

In the case of two or more dominating roots of equal modulus, the process does not converge, but a sequence can be derived which converges to the polynomial of the remaining zeros.

S. Moriguti, Japan

3819. **Theocaris, P. S., The curvilinear rosette method as an auxiliary device for the numerical solution of Laplace's equation,** *Proc. Soc. exp. Stress Anal.* **14,** 2, 89–96, 1956.

In the linear rosette method described by Frocht [title source, **VI,** 1, 39–43, 1948] for obtaining good initial values in solving Laplace's equation by numerical methods, straight lines are drawn through a given point. Each line is viewed as a taut string whose slope is determined by boundary values. The initial value is taken as the average ordinate at the given point. Author describes a similar method in which the taut strings are replaced by arcs of circles. If three ordinates, one in the interior and two on the boundaries, are known, the arc can be readily constructed. Likewise, if two ordinates and the tangent to the circle at one point are known, the circle can be constructed. Author shows how tangent may be determined. Examples given show some excellent estimations of initial values.

In the reviewer's opinion, the effort required for greater accuracy in the initial values by the method described may in many cases exceed that involved in additional iteration required, because less accurate initial values were used.

E. A. Ripperger, USA

3820. **Luke, Y. L., Simple formulas for the evaluation of some higher transcendental functions,** *J. Math. Phys.* **34,** 4, 298–307, Jan. 1956.

Author applies the modified trapezoidal rule to compute a certain class of definite integral. In numerous applied problems, this procedure permits further simplification of results obtained in closed form. Evaluation of multiparameter functions arising from indefinite integrals involving these transcendents is simplified. Computation is made easier as the technique requires evaluation of elementary functions usually available as a subroutine on an automatic computer. Computer coding is simplified and the need for punched card tables or table look-up is eliminated.

Author explores the trapezoidal rule developed by Poisson and others, and shows that the modified trapezoidal rule offers more simplicity than the trapezoidal rule, with virtually no loss of accuracy; it can be more accurate. Several examples are presented.

J. N. Aguirre, Argentina

3821. **Bolton, H. C., and Scoins, H. I., Eigenvalues of differential equations by finite-difference methods,** *Proc. Camb. phil. Soc.* **52,** 2, 215–229, Mar. 1956.

Approximations for eigenvalues of Sturm-Liouville differential equations under two-point boundary conditions are found by finite difference methods. The finite difference eigenvalues converge to the true eigenvalues as the mesh interval h is decreased, and as a rule these eigenvalues are less than the true values by quantities of order h^2. Neville's method of extrapolation, carried to zero h, is shown to give rapid accurate estimates of the true eigenvalues. The eigenfunctions are found by a suitable relaxation technique. Some examples from quantum mechanics are given.

A. F. Pillow, Australia

3822. **Diniak, A. W., and Weaver, E. R., A new computer for calculating the water content of gases,** *J. Res. nat. Bur. Stands.* **56,** 5, 269–278, May 1956.

A new computer designed to reduce the labor in computing the water content of gases is described, and details of construction and use are given. The computer is of the circular slide-rule type with an accuracy of about that of a 10-inch slide rule. The computer consists of three disks with appropriate circular scales, which automatically correct for all deviations from ideality. It is a completely new design of an earlier model, and it is based on newly determined values for the corrections. The primary function of the computer is to calculate the water content and dewpoint of gases at pressures up to 6000 psi quickly, accurately, and with a minimum of effort, but it may be used for many other computations. Attention should be called to the fact that deviations from ideality are not dependent on methods of measurement and that they affect

many measurements and computations in which they are usually ignored. From authors' summary

3823. Duffin, R. J., and Keitzer, E., Formulae relating some equivalent networks, *J. Math. Phys.* **35,** 1, 72–82, Apr. 1956.

3824. Saelman, B., Volume intercepted between two right circular cylinders, *Civ. Engng., N. Y.* **26,** 2, 61–62, Feb. 1956.

Book—3825. Stepanov, Yu. A., et al (edited by), Concise polytechnical dictionary [Krathiĭ politekhnicheskiĭ slovar], Moscow, Gosudarstvennoie Izdatelstvo, 1955, 1136 pp. $10.

This large volume contains short articles on various technical terms with clear sketches. Contributors attempted to avoid foreign names and terms. Surprisingly, many American terms have penetrated into the Russian language; they still use booster, carter, combine, compound, converter, conveyer, cracking, economizer, grader, impeller, mixer, offset, scraper, tubing, etc. Some articles are spoiled by superfluous tendency and dogmatism; e.g., hydraulics is described as contributing to the materialistic natural science, but not physics or mechanics! This technical encyclopedia will be useful for a large number of readers; this first edition is published in 200,000 copies. S. Kolupaila, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 3815, 3855, 3860, 3863, 3887, 4019, 4106, 4109, 4110, 4114, 4157)

Book—3826. Hertz, H., The principles of mechanics presented in a new form, New York, Dover Publications, Inc., 1956, 271 pp. $1.75 (paperbound).

This is a reprint (in the clear type usual with Dover publications) of the great posthumous classic of Heinrich Hertz. The book was published a year after the author's death in 1894 and it is here reprinted in an English translation with interesting prefaces by Hertz himself, by his teacher Helmholtz, by the colleague who edited the manuscript after his death, by the translator, and, last but not least, by Professor Cohen of Wesleyan University. The book itself is an exposition of mechanics in the Euclidian style, being a series of logical deductions not from axioms but from a few experimental observations. It reads like Euclid, short paragraphs carry the caption of Definition, Observation, Problem, Proposition, Corollary, etc. This book takes a worthy place among the Dover classical reprints, which include the works of Rayleigh, Lamb, Love, and Helmholtz.

J. P. Den Hartog, USA

Book—3827. Mestscherski, I. W., Problems in mechanics [Aufgabensammlung zur Mechanik] (Translated from Russian), Berlin, VEB Deutscher Verlag der Wissenschaften, 1955, 391 pp. DM 22.

According to the preface, book is intended for use at the Technical Universities in Russia. The approximately 1350 problems are divided into three groups, viz., statics, kinematics, and dynamics. Starting with the equilibrium of four forces, acting along the same axis, the first chapter deals with simple truss and cable problems, including friction of moving bodies, to be solved analytically and graphically; space forces and couples and spare frames; the determination of the center of gravity of plane sections and three-dimensional bodies.

The second chapter starts with problems on the kinematics of a point, followed by kinematics of plane motion (including gears, crank mechanisms, camshafts and eccentrics); more complicated movements (governors, rolling wheels, three- and four-bar linkages, etc.); rotation in space, including gyroscopes. The third chapter starts with problems on the determination of forces from a known motion and deals further with rectilinear and curvilinear motions; impulse, kinetic and potential energy, power; linear vibrations of single-degree-of-freedom systems and pendulums; impact; systems with variable mass (rockets, baloons, etc.); Lagrange equations; small vibrations of systems with two and more degrees-of-freedom (vibration-isolation, torsional vibrations of crankshafts, motions of vehicles, turbine foundations); problems of stability; etc.

The book contains a number of very interesting problems and should be suitable for graduate and postgraduate courses in mechanics. R. G. Boiten, Holland

3828. Jones, R. P. N., The orthogonal property—a proof by virtual work, *J. appl. Mech.* **23,** 2, p. 318 (Brief Notes), June 1956.

3829. Landweber, L., On a generalization of Taylor's virtual mass relation for Rankine bodies, *Quart. appl. Math.* **14,** 1, 51–56, Apr. 1956.

Paper presents a new derivation of Birkhoff's generalization of Taylor's formula on added masses which, furthermore, results in a simple and elegant interpretation of a term which Birkhoff had left as a surface integral.

As is well known, the kinetic energy of a rigid body can be expressed as a quadratic form of the six components of its motion, the coefficients of the said quadratic form M_{ij} being easily calculated from the mass distribution. If the body is moving through an inviscid incompressible fluid, the kinetic energy of the fluid is expressible as a quadratic form of the components of the rigid motion of the body; the coefficients A_{ij} are, in this case, to be evaluated as a surface integral containing the potential function. If the body is a Rankine one, Taylor's formula provides the coefficients A_{ii} $(i=1,2,3)$ in terms of the prescribed singularity distribution. Birkhoff generalized Taylor's results to the A_{ij}'s $(i=1,2,3,\cdots;\ j=1,\cdots,6)$. Lamb derives Taylor's formula by equating asymptotic expression for the potential, given first in terms of the added mass, and furthermore in terms of the singularity distribution. It is shown that Lamb's method can also yield Birkhoff's generalization, and, furthermore, that the term of Birkhoff's formula containing the surface integral is simply M_{ij}, the corresponding component of the mass tensor, for a body of the same form of the given one, and of density equal to that of the fluid.

The results do not give relations for all the added masses. The six terms due to pure rotation are not given. Attempts to find such relations have given numerous results which resemble, but are not quite, the ones sought. P. Santini, Italy

3830. Poulis, J. A., Poldervaart, L. J., and Teunissen, P., The measurement of small forces by means of a moving coil meter with an application to magnetic susceptibility measurements, *Appl. sci. Res. (B)* **6,** 1–2, 124–128, 1956.

A method is described for the measurements of small forces (10^{-5} – 10^{-3} newton) on a sample weighing a few grams. This sample is placed on a bar which is fixed on the turning coil of an ordinary switchboard current meter. The force on the sample can be compensated by the Lorentz force on the current through the coil. This method avoids a great many of the usual constructional difficulties of such apparatus and leads to a robust instrument which is easy to handle and to automatize, thus being of interest for use outside the laboratory. Especially, this possibility has induced the choice, as an example, of a magnetic susceptibility balance. For magnets of field strength up to 20,000 oerstedt, the precision obtained for the measured susceptibility amounts to 1/10%. From authors' summary

3831. Rosenauer, N., Application of complex variables for the synthesis of a linotype movement with prescribed limits of the

...tput angular velocity (in German), *Ing.-Arch.* 24, 1, 43–46, 1956.

The problem of finding the motion of a mechanism can be ...ated analytically by the use of complex variables, as described ...earlier literature. Author solves the problem of finding the two ...sitions of a four-link chain in which the output angular velocity ...minimum and a maximum, respectively.

C. B. Ludwig, USA

3832. **Kloomok, M., and Muffley, R. V., Determination of radius of curvature for radial and swinging-follower cam systems,** *Trans. ASME* 78, 4, 795–802, May 1956.

Equations are derived for pressure angle and cam curvature and ...ts are presented indicating quick solutions of numerical ...blems. Equations are longer than those previously derived by ...ter [*Trans. ASME* 70, 483–489, 1948] but were derived with ...line computation in mind.

A. S. Hall, USA

3833. **Kloomok, M., and Muffley, R. V., Determination of pressure angles for swinging-follower cam systems,** *Trans. ASME* 78, ...03–806, May 1956.

See preceding review.

3834. **Webb, C. R., The effect of gradient on fuel consumption and speed of a road vehicle,** *Instn. mech. Engrs. Auto. Div.* Part ...101–117, 1952–1953.

There is a considerable need for a reliable means of determin... the effect of gradient on fuel consumption and journey time of ...l vehicles. In order to decide whether a proposed road ...provement, involving the elimination of steep gradients, is to be ...conomical proposition, some reliable estimate is necessary of ...l and time likely to be saved as a result. The approach used ...f general application, and depends on equating the engine out... to the resistance to motion under uniform speed conditions. ...It is concluded that, for a light vehicle, a nearly level surface ...best from the economy point of view. Fuel consumption for a ...ney between two points at the same altitude, on roads sloping ...one in four, is nearly three times that for the same journey on ...level. At the same time, the journey takes over three times as ...

The paper also reveals the inherent inefficiency of the spark-...ition engine used, since, for the majority of journey times, it is ...part-throttle operation. Arguments are made for wider gear ...ios and improved engine characteristics on private cars.

From author's summary

Servomechanisms, Governors, Gyroscopics

(See also Revs. 3815, 3908, 4015, 4054)

Book—3835. **Laning, J. H., Jr., and Battin, R. H., Random processes in automatic control,** New York, McGraw-Hill Book Co., Inc., 1956, ix+434 pp. $10.

Book treats the mathematical problems concerning the analysis ...d selection of control system having random input.

The treatment of statistical fundamentals occupies about half of ...e book. After an introductory chapter describing the general ...ture of the problem and scheme of the book, the next two ...apters deal with basic principles of probability theory and sta...tical description of random process. Then follows a chapter on ...e application of the basic concepts to the study of shot effect ...d general Gaussian random process.

Actual treatment of the application to control systems begins ...with chap. 5, which deals with time-invariant linear system on ...stationary random process. Here the input-output relation of power spectral densities is derived and methods of computing mean-squared errors are explained.

Chapter 6 presents some interesting and original methods of dealing with error analysis of nonstationary problem, including the method of adjoint systems. Chapter 7 explains the Wiener optimizing problem for stationary process and gives a few methods of solving the Wiener-Hopf equation. The last chapter presents a systematic approach to the optimization problem using finite data.

The treatment is concise and yet clear and complete. Mathematical development, sometimes deleted from main text to avoid disrupting the continuity, is thorough and accurate. There are many examples well worked out to help the understanding. Reviewer feels that authors' objective for an organized exposition of the fundamentals of the theory of random process and of the techniques for the application to automatic control problems, is well accomplished.

L. S. Dzung, Switzerland

Book—3836. **Letov, A. M., Stability of nonlinear regulated systems [Ustoichivost nelineinykh reguliruyemykh sistem],** Moscow, Gos. Izdat. Tekh.-Teor. Lit., 1955, 312 pp. 9.20r.

This fine monograph deals with problems of absolute stability. It extends fundamental results obtained by A. I. Lur'e to more complicated types of control systems. Its aim is to familiarize interested specialists with various applications of Lyapunov's direct method in solving important problems of automatic regulation.

Book opens with a brief introduction on the necessary facts from the stability of motion, as developed by A. M. Lyapunov. There are 11 chapters, a brief nominal index, and a list of matters treated.

Chapters I–III are introductory in character, giving the statement of basic problems on absolute stability and transforming fundamental equations to various canonic forms used for starting point in treating concrete cases. Following two sections are devoted to Lyapunov's results on strictly stable systems with one regulating member, and to various simplified stability criteria of such systems. Then come two methods (due to A. I. Lur'e and to the author) for studying strictly unstable control systems. Chapter VIII shows two ways of estimating the degree of stability of regulating systems with one member. Absolute stability in the case of two-member systems is treated in chap. IX, which is followed by considerations of the stability under constantly acting disturbances. The last section gives theoretical statement and solution of the stability problem in the case of unsteady motion of regulating system.

Paper is good, print careful. There are 12 instructive figures throughout the text. The clearness of attractive theoretical considerations is enhanced by numerous examples.

The book must be warmly recommended to engineers interested in theory and applications of servomechanisms.

V. Vodička, Czechoslovakia

Book—3837. **Herschel, R., Laplace transformation in automatic control [Laplace-Transformationen in der Regelungstechnik],** München, R. Oldenbourg, 1955, 142 pp. DM 12.8.

This book is a collection of lectures given by outstanding German authorities in the field of automatic controls. These lectures are edited by R. Herschel to a well-rounded booklet. Scope is indicated by the title of the eleven lectures: (1) W. Oppelt: Deduction of the control equations of a control system. (2) G. Doetsch: Introduction to the Laplace transformation. (3) R. Herschel: Exercises in the Laplace transformation with examples. (Single equations.) (4) St. Schottlaender: Exercises in the Laplace transformation with examples. (System of differential equations.) (5) J. Dorr: Exercise in the Laplace transformation with a complicated example. (6) A. Leonhard: The method of frequency analysis shown on a regulating system example. (7)

O. Schafer: The Laplace transformation applied to a regulating system example. (8) K. Kupfmuller: Regulations with time delay. (9) R. C. Oldenbourg: The use of Laplace transformation for piecewise linear regulating systems. (10) H. F. Schwenkhagen: Interpretation of the Laplace transformation. (11) T. Peters: Treatment of the regulating circuit with the methods of information theory.

Reviewer found the large collection of excellent examples a very useful exercise. The book is recommended to both students and graduate engineers seeking good and advanced demonstrations of systems analysis theory. Dr. Oldenbourg's lecture is particularly interesting as an introduction to nonlinear analysis of control systems. P. P. Biringer, Canada

3838. Nightingale, J. M., A basic outline of servo mathematics, *Mach. Design* **28,** 13, 74–81, June 1956.

3839. Tagajewskaja, A. A., The determination of the amplitude-phase characteristic of linear systems from a transient response (in German), *Regelungstech.* **4,** 3, 68–72, 1956.

The response of a regulator, servo, or other linear system to a step function being given, it is desired to obtain the amplitude-phase characteristics (for a servo, the open-loop transfer function). This is given by a Laplace transform. Author replaces the transient by equidistant ordinates and expresses the amplitude corresponding to a given angle by an infinite series. He gives complete formulas suitable for responses of three different types, and derives them in an appendix. The complete numerical computation of an example is given. A very conscientious paper, which should be consulted by computers.

P. LeCorbeiller, USA

3840. Teasdale, A. R., Jr., Get frequency response from transient data by adding vectors (Two ways to get frequency response from transient data). Method no. I, *Control Engng.* **2,** 10, 55–59, Oct. 1955.

Paper outlines a graphical method whereby frequency-response characteristics of a system may be derived from the measured transient response to a known input. Examples include inputs of a unit step, a unit impulse, and a unit doublet impulse. Procedure appears to be quite practical for engineering use. With standard drafting instruments, data could be reduced in a small fraction of time required to obtain the transients, the whole procedure being much shorter than direct measurement of frequency response.

Paper combines mathematical abstractions with graphical practices in a somewhat unfortunate fashion. Less mathematics and a more clearly defined graphical routine would help the engineer, while more thorough treatment of the mathematics with less comprehensive examples and diagrams would make paper more acceptible to the physicist or mathematician. Basic ideas appear to be quite sound. Criticism pertains to areas of emphasis only. C. D. Pengelley, USA

3841. Reynolds, J. G., Jr., Get frequency response from transient data by machine computing (Two ways to get frequency response from transient data). Method no. II, *Control Engng.* **2,** 10, 60–63, Oct. 1955.

Paper outlines the features of an electric analog computer necessary to obtain frequency-response characteristics of a system from a measured transient response to a unit impulse input. The majority of the paper is devoted to a particular design and its application to particular problems. As is true of most electronic computer devices, it is valuable only when a large number of similar operations must be made; otherwise equipment cost is prohibitive. C. D. Pengelley, USA

3842. Peters, J., Contribution to the study of dynamical behaviour of an instrument, *Rev. Tijdschr.-Mecan. Werk.* 1, 5, 34–43, Oct. 1955.

3843. Paul, S., Thermal feedback as a supplement of two-step control (in German), *Regelungstech.* **3,** 12, 296–302, 1955.

Theoretical and experimental study. For a sufficiently inert plant the thermal feedback converts the discontinuous control process into one which is practically continuous.

I. Flügge-Lotz, USA

3844. Solodovnikov, V. V., and Matveev, P. S., Synthesis of servomechanism correctors in the presence of disturbances traced to fixed requirements for dynamic precision (in Russian), *Avtomatika i Telemekhanika* **16,** 3, 233–257, 1955.

A method is worked out which allows synthesis of correcting devices to be added to a servo system with fixed transfer function, the system acted upon by signal and noise (stationary random function), so as to minimize the rms-error for given system response time and error coefficients.

The paper contains tables and nomograms to aid the synthesis work and also gives some examples which show the practical applicability of the method. H. Christensen, Norway

3845. Macphee, J., How to control a boiling reactor, *Nucleonics* **13,** 12, 42–45, Dec. 1955.

Paper explores some of the problems of controlling a boiling-type nuclear reactor coupled to a power plant. Details of mechanical features of control systems are not discussed but rather the manner in which the power plant or load affects the reactivity of the reactor. Since the feedback mechanisms are rather complex, author discusses in some detail a model of a boiling reactor for the case of constant-pressure operation.

R. S. Wick, USA

3846. Johnson, B. A., Schmid, L. D., and Warshawsky, J., What to look for in electrohydraulic servo valves, *Control Engng.* **3,** 6, 84–96, June 1956.

Hydraulic servo systems offer considerably faster dynamic response than electric or pneumatic systems, particularly in applications requiring more than one horsepower at the load. This response comes from the high torque-to-inertia ratio of the servo's hydraulic actuator and additional advantages come from the inherently large power-to-weight and power-to-volume ratios of the actuator; wide bandwidth and large power-handling capacity of the servo's electrohydraulic control valve.

Table I reviews the major elements and forms of electrohydraulic servo control valves, valves that provide the system designer with effective components for severe control problems. This article treats the important design considerations in selecting these valves for system application. From authors' summary

Vibrations, Balancing

(See also Revs. 3908, 4137, 4150, 4152)

3847. Clauser, F. H., The behavior of nonlinear systems, *J. aero. Sci.* **23,** 5, 411–434, May 1956.

Author has set himself the task of promoting an understanding of nonlinear phenomena by presenting them "in terms of fundamental and well-understood concepts." This is an extremely laudable effort and author should be credited with having accomplished this difficult task, in general, very well.

Treated are (1) systems capable of auto-oscillations (with particular emphasis on the phase plane concept); (2) forced oscillations in systems not capable of auto-oscillations, where the so-called jump phenomenon and the subharmonics are considered;

3) forced oscillations in systems capable of auto-oscillations, where, in particular, the entrainment of frequencies is considered; (4) the behavior of systems with varying coefficients; and 5) systems described by partial differential equations; here some particularly elucidating remarks are made.

In a field which is still in rapid evolution and where concepts have not yet been formulated to complete satisfaction, an expository paper is burdened with a particular responsibility: (a) to help such an evolution; (b) not to do anything which endangers the process. Although reviewer thinks that author has done a very creditable job, there are a few items where he feels that author violated stipulation (b) because of either unpermitted generalization or improper terminology. However, because these points are more or less subtle ones and should not distract from the fine accomplishments, their discussion will be left to another occasion (presumably: Readers' Forum, title source).

K. Klotter, USA

3848. Kats, A. M., Forced vibrations of nonlinear systems with one degree of freedom and nearly conservative (in Russian), *Prikl. Mat. Mekh.* **19,** 1, 13–32, 1955.

A nonlinear vibration defined by the differential equation

$$\ddot{x} + F(x) = \epsilon f(\dot{x}, \ddot{x}, \epsilon, t)$$

is investigated. Assume that $x_0 = x_0(t + \tau, \omega)$ is a general solution of $\ddot{x}_0 + F(x_0) = 0$ and

$$x = x_0 + \epsilon x_1 + \epsilon^2 x_2 + \cdots$$

$$F(x) = F_0 + \epsilon F_1 + \epsilon^2 F_2 + \cdots$$

$$f(\dot{x}, \ddot{x}, \epsilon, t) = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots,$$

then we can get a system of linear differential equations by substituting them into given equation.

If we put

$$F_{n+1} - F'(x_0)\, x_{n+1} = \Phi_n(x_0, x_1 \cdots x_n)$$

$$f_n - \Phi_n = g_n$$

periodic solutions can be obtained when

$$\int_0^T f_0 \dot{x}_0 dt = 0, \int_0^T g_1 \dot{x}_0 dt = 0 \cdots, \int_0^T g_n \dot{x}_0 dt = 0$$

The author has proved that if $a = \max x_0$ and

$$\frac{d\omega}{da}\frac{\partial}{\partial \tau}\int_0^T f_0 \dot{x}_0 dt < 0, \int_0^T \frac{\partial f e}{\partial \dot{x}_0} dt < 0$$

the vibration is stable, and when

$$\int_0^T f_0 \dot{x}_0 dt = 0 \cdots, \int_0^T g_k \dot{x}_0 dt = 0,$$

the condition for stability becomes

$$\frac{d\omega}{da}\lim_{\epsilon \to 0}\left(\frac{1}{\epsilon k}\int_0^T f \dot{x}_0 dt\right) < 0, \lim_{\epsilon \to 0}\left(\frac{1}{\epsilon^e}\right)\int_0^T \frac{\partial f}{\partial \dot{x}_0} dt < 0.$$

Finding τ from these equations, x_n can be found from the system of differential equations by the method of constant variation.

M. Kataoka, Japan

3849. Drozdov, Yu. M., Forced vibrations of nonlinear systems with one degree of freedom and nearly conservative; examples (in Russian), *Prikl. Mat. Mekh.* **19,** 1, 33–40, 1955.

Three examples of Kats method [see preceding review] are illustrated. When $F(-z) = -F(z)$ and $x_0(z) = \sum_{p=0}^{\infty} A_{2p+1} \cos(2p + 1)$, where $\omega(t + \tau) = z$,

(1) the vibration defined by

$$\ddot{x} + F(x) = \epsilon(p \cos m\omega t - \beta \dot{x})$$

is stable when

$$\sin 3\theta = -\frac{\beta\omega}{3PA_3}\sum_{n=1}^{\infty}(2n-1)^2 A_{2n-1}^2 > 0, \cos 3\theta > 0$$

(2) The vibration defined by

$$\ddot{x} + F(x) = \epsilon P \cos 2\omega t - \epsilon^2 \beta \dot{x}$$

is stable when

$$KH_2^{(-2)} \cos 4\theta > 0.$$

(3) The vibration defined by

$$\ddot{x} + F(x) = \epsilon x P \cos \omega t - \epsilon^2 \beta \dot{x}$$

is stable when

$$KA_1^3(4A_1^1 - A_1)\cos 2\theta > 0,$$

where $\theta = \omega\tau$

$$H_2^{(-2)} = \tfrac{1}{2}(3A_1A_3 + 3A_3A_1' - A_1A_3')$$

$$\frac{1}{K} = \frac{\omega^2}{2}(A_1^2 + 2A_1A_1')$$

Numerical examples for the case where $F = x + x^3$ are given.

M. Kataoka, Japan

3850. Pailloux, H., Lateral vibration of a loaded beam (in French), *C. R. Acad. Sci. Paris* **242,** 17, 2097–2099, Apr. 1956.

The differential equation and the boundary conditions for the problem described in the title are derived from Hamilton's principle.

B. A. Boley, USA

3851. Numachi, F., Suppression of pump vibrations set up at starting up—preopening method, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55-A-145, 10 pp. + 8 figs.

Cases occur in large water pumps wherein, after installation, vibrations exceeding allowable limits are found to set in when starting up. As a measure to meet such situations without resorting to reconstruction of the pump itself, a method of "preopening" is presented which consists of opening up the delivery valves to a certain extent while the pump is gradually being worked up to full speed. The method of estimating the quantity of reverse flow, the torque required, and the time necessary for attainment of normal speed is explained. An example of application in actual practice is given.

From author's summary

3852. Söhngen, H., Vibrational behavior of a blade ring in vacuum (in German), *Dtsch. Versuchsanstalt Luftfahrt E. V. Rep.* 1, 24 pp., Aug. 1955.

The vibrations of blade rings of axial compressors or turbines are investigated and determined. It is assumed that the modes and frequencies of free vibrations of a single blade are known, and, further, that the displacements of neighboring blades are coupled through the ring. With these assumptions, the analysis is made of the natural, damped, and forced modes and frequencies of vibrations of the blade ring in vacuum. The following cases are examined: (a) all the blades have the same frequency; (b) the blades have alternately two different natural frequencies.

W. Ornstein, USA

3853. Bray, A., Vibration and acceleration measurement (in Italian), *Ricerca* **6,** 1, 41–56, Jan.–Mar. 1955.

3854. Zweifel, O., Calculation of the elastic behavior of railway vehicles in free vibration. Parts I, II (in German), *Schweiz. Bauztg.* **74,** 1, 1–6, Jan. 7, 1956; **74,** 2, 17–22, Jan. 14, 1956.

3855. **Smiley, R. F., Correlation, evaluation, and extension of linearized theories for tire motion and wheel shimmy,** *NACA TN* 3632, 139 pp., June 1956.

Paper examines stability of motion of a single rubber-tired wheel with flexible mounting, allowing for five degrees of freedom. These are: swivelling about vertical axis, tilting, horizontal displacements along and perpendicular to direction of travel, lateral displacement of tire footprint due to tire distortion. Differential equations of motion are formed for these variables and methods for assessing stability are described. Several earlier theories are shown to be derivable from present treatment by introducing approximations.

Account is very complete and would be most valuable to anyone faced with a particular shimmy problem.

D. C. Johnson, England

3856. **Anonymous, Calculating damping factors for dashpot dampers,** *Prod. Engng.* **27**, 4, 162–165, Apr. 1956.

Paper presents straightforward elementary calculation.

J. H. Greidanus, Holland

Wave Motion in Solids, Impact

(See also Revs. 3914, 3926, 3937, 4135)

3857. **Pekeris, C. L., Solution of an integral equation occurring in impulsive wave propagation problems,** *Proc. nat. Acad. Sci. Wash.* **42**, 7, 439–443, July 1956.

Traditional method of treating wave propagation caused by a sudden application of a force is first to obtain a solution for a periodical source and then to generalize it through Fourier synthesis. But this method is suitable for obtaining the solution only at great distances. Close to the source a "ray theory" must be used. In this case, an integral equation appears in which a Bessel function and a radical of the form $(x^2 + a_k^2)^{1/2}$ are involved. Such type of integral equations are found in many problems of wave propagation in which the media are doubly or multiply refracting. Author gives the list of such problems.

A general method of solving such an equation was recently given by the author, and, taking the example of doubly refracting medium, a summary is prepared here. Nature of the solution differs in the case:

(1) Velocity of the radiated waves is larger than that of the other sort of wave. In this case, total reflection does not occur and the problem is relatively simple.

(2) Reverse case of (1): (a) Region near the origin, in which total reflection does not occur; (b) Region where the total reflection occurs.

In every one of these cases the solution is given in the form of definite integral over the range from 0 to $\pi/2$. This is convenient for evaluation by an electronic computer.

A very useful summary for those who are engaging in the study of wave propagation. A good reference is given.

Y. Sato, Tokyo

3858. **Davies, R. M., Stress waves in solids,** *Brit. J. appl. Phys.* **7**, 6, 203–209, June 1956.

The aim of this paper is to discuss, in fairly elementary fashion the topic of the transmission of transient stress pulses of short duration in solids, together with the associated problem of the measurement of pressures subject to very rapid changes. The pressure bar method, devised by Betram Hopkinson, forms one of the basic experiments in the field and it is described in some detail. An analysis of the limitations of the method leads to a discussion of the change of form of stress wave pulses as they recede from the origin, and the main features of the distortion are discussed when the pulses are propagated in (a) elastic solids, (b) viscoelastic solids, and (c) solids stressed beyond the elastic limit.

From author's summary

3859. **Herrmann, G., Influence of large amplitudes on flexural motions of elastic plates,** *NACA TN* 3578, 45 pp., May 1956.

Starting with the fundamental equations of the general three-dimensional nonlinear theory of elasticity for the case of small elongations and shears but moderately large rotations, a set of plate equations of motion is derived for an isotropic material obeying Hooke's linear law. Certain assumptions as to plate displacements are made at the outset. The resulting nonlinear plate theory of motion, valid for large deflections, is discussed in the light of the three-dimensional theory and other nonlinear plate theories; in particular, the static von Karman plate theory.

In order to investigate the influence of large rotations in a dynamic problem, the nonlinear equations are solved for the case of propagation of straight-crested waves, and the wave velocities are computed for various values of the parameters involved. Certain similarities to free vibrations of a mass on a nonlinear spring, governed by Duffing's equation, are established.

In addition, a variant of plate equations is derived which permits tracing back the origin of certain terms appearing in other nonlinear plate theories, thus revealing more clearly the underlying assumptions of those theories.

From author's summary by M. V. Barton, USA

3860. **Lightfoot, E., Shock loading effects in overhead travelling cranes,** *Engineering* **181**, 4700, 169–173, Apr. 1956.

Elasticity Theory

(See also Revs. 3877, 3886, 3888, 3891, 3892, 3897, 3899, 3901, 3902, 3907, 3931, 3947, 4147)

Book—3861. **Raskovic, D., Strength of materials [Otpornost Materi'ala],** Belgrade, Universiteti Beogradi, 1955, xii + 300 pp.

This is an undergraduate textbook on strength of materials covering the conventional subject matter for engineering students. It contains the following chapters: Introduction (analysis of stress and strain); 1. Axial (one-dimensional) stress; 2. Two-dimensional stress; 3. Shearing; 4. Three-dimensional stress; 5. Torsion; 6. Pure bending; 7. Bending by forces; 8. Skew bending; 9. Elastic lines; 10. Statically undetermined problems; 11. Curved rods; 12. Buckling; 13. Compound stresses; 14. Bending of circular plates; 15. Stresses due to dynamic forces; Appendix (moments of inertia).

The book has no problems. Author refers to his "Collection of problems on strength of materials and tables," Beograd, 1955 [AMR 8, Rev. 3320]. It seems to be a good textbook, but in reviewer's opinion it is inferior to the best American textbooks on the subject.

T. Leser, USA

3862. **Gutstein, W. H., A generalization of Hooke's law in muscle elasticity,** *Bull. math. Biophys.* **18**, 2, 151–170, June 1956.

Experimental tension-length (stress-strain) curves on muscle fibers are reviewed with reference to their a posteriori implication concerning the failure of Hooke's law to hold. Consideration of the true stress as a tensor point function in connection with the classical laws of elasticity allows for a further generalization of Hooke's law. This generalization when restated in terms of the coordinates, which are equivalent to those used in the experimental data, results in curves which predict the data in a qualitative and semiquantitative manner.

The treatment is phenomenological with respect to elastic properties of muscle fibers and is developed without reference to

thermo elastic or myophysiologic properties.

From author's summary

3863. **Teodorescu, P. P., Method to solve two-dimensional problems in case of arbitrary mass forces** (in Rumanian), *Acad. Repub. pop. Rom. Comun.* **6**, 2, 285–290, 1956.

For various laws of loading, the respective equations governing the stress function are presented, as well as those for finding stresses by this function. Displacements also are given, along with suggestions for applying method to deep beams of rectangular cross section with different boundary conditions; also for concentrated forces acting in the interior of beam.

From author's summary by W. Mudrak, Austria

3864. **Prager, W., On conjugate states of plane strain,** *J. Mech. Phys. Solids.* **4**, 3, 167–171, May 1956.

Author proves an important and useful duality principle for pairs of incompressible solids (not necessarily linearly elastic) such that the relation between the stresses and the strains in one solid has the same mathematical structure as the relation between the strains and the stresses in the other. In this duality principle, which is valid for plane strain only, the forces and stresses in the one solid correspond to the displacements and strains of the other solid. An example is given in which a Maxwell solid corresponds to a Voigt solid. This principle is an extension of a dual correspondence between states of plane strain in an incompressible elastic solid described by Hill [AMR **9**, Rev. 1732].

A. Phillips, USA

3865. **Kharbanda, O. P., An improved nomogram for the ratios of octahedral to maximum shearing stresses,** *J. appl. Mech.* **23**, 2, p. 312 (Design and Data Methods), June 1956.

3866. **Ericksen, J. L., Stress deformation relations for solids,** *Brit. J. Phys.* **34**, 2, 226–227, Feb. 1956.

The author obtains a theory of finite elastic strain intermediate in generality between those of Cauchy's type and those of Green's type. The assumptions are: (1) the stress depends only on the displacement gradients (Cauchy's assumption); (2) arbitrarily near to any deformation, there is another which cannot be reached through a motion in which the stresses do no work. Assumption (2) is new in the theory of elasticity. From it and from Carathéodory's theorem on Pfaffian forms, the author reduces the expression for the stress (1) to the form

$$t_{ij} = J^{-1}\,\phi\, x_{i,\alpha}\,\partial\psi/\partial x_{j\alpha}$$

where the $x_{j,\alpha}$ are the displacement gradients, J is their Jacobian, and ϕ and ψ are scalar functions of the nine $x_{i,\alpha}$. Green's theory results as the special case when $\phi = \psi$ (ψ). Of greater interest than the generality of the result is the entirely new type of argument, based solely on considerations of mechanical work and independent of thermodynamics.

C. Truesdell, USA

3867. **Lee, L. H. N., Non-uniform torsion of tapered I-beams,** *J. Franklin Inst.* **262**, 1, 37–44, July 1956.

Investigation is made of the effects of bending and torsion of the two flanges of the beam, and effect of Saint-Venant torsion only of the web. From equation of rotational equilibrium $T = T_S + T_m + T_v$ where T is the applied torque at the free end, T_S the Saint-Venant torque, T_m = sum of component of bending moments in the flanges, T_v shear couple developed by bending of the flanges, and ϕ the angle of twist, a third-order differential equation in ϕ is obtained. A descending power series solution of the equation has been obtained to give the bending moment in the flanges. Since the ratio of web to flange thickness is high, the web deformation is neglected and twisting of the flanges and the web through the same angle has been assumed. The result is illustrated by three numerical examples.

D. N. Mitra, India

3868. **Reissner, E., Note on torsion with variable twist,** *J. appl. Mech.* **23**, 2, p. 315 (Brief Notes), June 1956.

See AMR **9**, Rev. 1738.

3869. **Jennings, J., Angle sections in bending determination of limiting design stress,** *Engineering* **181**, 4712, 558–559, June 1956.

3870. **Rivlin, R. S., Plane strain of a net formed by inextensible cords,** *J. rational Mech. Analysis* **4**, 6, 951–974, Nov. 1955.

Net considered is plane and consists of two families of parallel, perfectly flexible cords, joined at points of intersection. Author states the net is an idealization of certain types of textile fabric. The analysis, developed by treating the net as a continuum, established special results for homogeneous deformation and shows that equations governing the displacements in general plane deformation are of the hyperbolic type. Solutions are discussed for various types of boundary conditions. Equations of motion, boundary conditions, and stress deformation relations are discussed. The process of finding deformations for a net of arbitrary edge contour is outlined for the condition of specified edge transactions.

From author's summary by R. E. Newton, USA

3871. **Dorogobed, A. S., Pure shear-induced stress distribution in an orthotropic plate with a circular aperture** (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* no. 21, 113–119, 1955.

Author considers the stress concentration in the neighborhood of a circular cutout of a large rectangular plate of orthotropic material. Following the work of S. G. Lechnitski, the problem is reduced to the determination of two functions of two complex variables. The problem is resolved by the use of conformal mapping which essentially yields the form of the analytic functions. The stresses, that is the stress tensors, are given as functions of the analytic functions. Numerical examples are given by inserting the appropriate elastic constants into the derived formulas, and the resulting stress distributions about the cutout are graphically demonstrated. Much work of this nature has already been done by G. N. Savin and his co-workers, and the essentials are summarized in Savin's book on stress concentrations near cutouts.

J. J. Brandstatter, USA

3872. **Sokolowski, M., Orthotropic plates subjected to body forces** (in Polish), *Arch. Mech. stos.* **7**, 4, 427–456, 1955.

The subject of the paper is a plane state of stress (or strain) in an orthotropic plate (a "disk") subjected to arbitrarily distributed internal forces.

In both Cartesian and polar coordinates, a nonhomogenous quasi-biharmonic differential equation for the stress function is derived which constitutes a generalization of the Airy function. In this way a fuller analogy of E. E. Wieghardt is obtained between the problem of bending of a thin plate and that of a plate subjected to forces acting in its plane.

Using this analogy, author applies some methods known from the theory of thin plates in order to determine stress components in rectangular and in circular plates. In particular, it is pointed out that Fourier single and double series and integrals can be used to solve problems of a rectangular plate, of a strip, and of an infinite plate.

Stress components in a cylindrically orthotropic circular plate, subjected to gravity load and resting freely on a rigid foundation, are also determined. Differential equation for generalized Airy function and for the problem of a plate having slightly variable thickness is derived and one example is solved.

Z. Kaczkowski, Poland

3873. **Schneider, P. J., Variation of maximum thermal stress in free plates,** *J. aero. Sci.* **22**, 12, 872–873 (Readers' Forum), Dec. 1955.

A succinct resume is presented for stresses in thin, free plates under six types of heating. The equation of Timoshenko for thermal stress in a plate ["Theory of elasticity, McGraw-Hill Book Co., Inc., New York, 1934] is used to find the maximum stresses which result from the critical temperature distributions. The six cases examined include four different types of heat generation in solid plates and two types of coolant heat transfer in porous plates. The results are compared and discussed in detail.

H. Becker, USA

3874. Melan, E., Thermal stresses during the cooling of a sphere (in German), *Öst. Akad. Wiss. math.-nat. Kl. Anz.* **92,** 11, 202-224, 1955.

The transient thermoelastic stresses in a sphere initially at a uniform temperature and cooling to an ambient at constant temperature through a boundary conductance (Newton's law of cooling) are determined. A displacement potential formulation is used and a particular solution is found by the method of Goodier [*Phil. Mag.* **23,** p. 1017, 1937]; the required complementary solution to produce traction-free boundaries is then determined readily, since it corresponds to a uniform hydrostatic pressure.

J. H. Weiner, USA

3875. Nowiński, J., Thermal stresses in a thick-walled spherical vessel of transversally isotropic material (in Polish), *Arch. Mech. stos.* **7,** 3, 363-374, 1955.

Paper deals with the determination of strain and stress distribution in a thick-walled spherical container of transversally isotropic material, both elastic and thermal, in respect to the radii of the sphere. It is assumed that the temperature distribution is spherical.

The adopted method of solving the problem is to express the stress components in terms of radial displacement. The condition of internal equilibrium permits deriving a differential equation (of the second-order, Eulerian type) for the unknown radial displacement.

The general integral of this equation is used to determine stress components, boundary conditions being established for the case of thermal load only.

The following examples are considered: (1) a thick-walled spherical container (a hollow sphere) with temperature distribution corresponding to a steady heat flow; (2) a thin shell; this case has made it possible to pass to the problem of thermal stresses in a transversally isotropic plate fixed at the edges and heated on one surface; (3) a solid sphere.

The paper is illustrated by a numerical example.

W. Urbanowski, Poland

3876. Nowiński, J., Olszak, W., and Urbanowski, W., On thermoelastic problems in the case of a body with an arbitrary type of curvilinear orthotrophy (in Polish), *Arch. Mech. stos.* **7,** 2, 247-264, 1955.

Purpose of paper is to define stress and strain distribution in a body with an arbitrary type of orthotropy, in the case when orthogonal curvilinear coordinates have been chosen, such that the elements which are bounded by three pairs of respective coordinate surfaces are elastically identical at each point of the body; the above assumption concerns thermal conditions, also.

Furthermore, it is assumed that at each point of the body there are three planes of elastic symmetry, the main directions of elasticity (orthogonal with respect to each other) being, respectively, tangential to the lines of the coordinate system at the given point. Under these assumptions a rather wide class of important technical problems may be considered.

Paper comprises three parts: introduction, general considerations, and applications.

Authors give an expression for thermal stress as the sum of two components, $\overline{\sigma}_i$ and σ^*_i. The first is determined similarly as when solving problems of the theory of elasticity, with the additional condition that fictitious body forces and fictitious normal surface forces are introduced to Navier's equations and to boundary conditions; the second term $\sigma^*_i = -\alpha E T / 1-2\nu$ corresponds to the hydrostatic pressure.

In the special case of elastic and thermal isotropy and in the the case of rectangular Cartesian coordinates, the general equations that have been found are identical with the equations of the classical theory of elasticity.

Paper is completed by three examples. The first one deals with an aeolotropic thick-walled circular cylinder with arbitrary symmetrical distribution of temperatures, the length of the cylinder being assumed to be infinite.

The second example deals with the problem investigated in the first example but replacing the thick-walled tube by a thin circular ring.

As the third example, a thick-walled spherical container with transversal isotropy is taken into consideration.

Z. Klebowski, Poland

Experimental Stress Analysis

(See also Revs. 3858, 3899, 4148, 4149)

3877. Gerard, G., and Gilbert, A. C., Photo-thermoelasticity: an exploratory study, New York Univ. Res. Div., TN Rep. SM 56-11 (AFOSR-TN-56-237), 24 pp. + 18 figs., May 1956.

This report presents a very useful method of studying two-dimensional thermoelastic problems by photoelastic methods. Thermocouples are buried in, or attached to, the photoelastic model and then temperature gradients are established by subjecting the surface of the model to refrigeration; for example, contact with dry ice. Several photoelastic materials were investigated and it was found that Paraplex had the best over-all characteristics for this work. Photoelastic studies were carried out for the plane stress problems of a disk contracting upon an elastic inclusion and the transient thermal stress field produced by a temperature differential suddenly applied to the upper edge of a long beam. Excellent agreement between theoretical and experimentally determined results was obtained.

E. E. Sechler, USA

3878. Teague, J. M., Jr., and Blau, H. H., Investigations of stresses in glass bottles under internal hydrostatic pressure. Part I, Photoelastic studies of Fosterite models; Part II, Breakage studies of glass bottles under internal hydrostatic pressure; Part III, Electric strain gauge and brittle coating studies, *J. Amer. ceram. Soc.* **39,** 7, 229-252, July 1956.

Report describes extensive investigation of stresses and bursting pressures. Stresses were determined by frozen-stress photoelastic techniques carried out on Fosterite plastic models and with resistance gages on inner and outer surfaces of glass bottles. The maximum stresses occurred at the inner radius between bottom and side wall. This stress was 2 to 2½ times the stress in cylindrical wall. High stresses also occur at the outside center of the bottom. Areas of stress concentration were identified with brittle lacquer coatings. These coincided with results of stress analysis. Bottles grit-blasted over inner or outer surfaces were hydrostatically tested to failure. Failure sources coincided generally with stress areas shown by brittle lacquers. The purpose of grit-blasting was to bring the glass surfaces to a relatively uniform strength so that fracture origins would occur at points of highest stress.

Other bursting tests were made on bottles (1) not handled after fabrication, (2) given severe handling abuse. Comparison showed that surface abuse reduced average bursting pressures by as much as one third.

Authors conclude that all results of these tests can be correlated satisfactorily when various factors are considered. Failures under pressure seldom originate at point of maximum stress (lower inside radius). This surface is of high initial strength and is protected from abuse. After handling or grit-blasting, most fractures originated in the side walls of the bottles.

E. B. Shand, USA

3879. **Katsuhiko, I., Studies on photoelasto-plastic mechanics, on the yielding and strain figure of ductile linear polymer solids,** *J. sci. Res. Inst. Tokyo* **50,** 19–28, Mar. 1956.

Celluloid is used as representative ductile linear polymer. It has an upper yield point, at which fine strain figures appear in 45° direction to the tension. Elongation decreases with increase rate of strain.

Yielding under plane stress is studied on a plate with circular hole under uniaxial tension. The strain figures in the neighborhood of the hole are similar to the isochromatics in a circular polariscope. The boundary line at which the strain figure starts in the yielding is between the isochromatics of 3 and 4 fringe order. With increasing load, strain figures propagate in the same manner as the isochromatics.

H. Mussmann, Germany

3880. **Hiltscher, R., Minimum-weight design of structural components, with help of photoelasticity** (in German), *Z. Konstr.* **7,** 211–219, 1955.

Author develops method for minimum-weight design of structural components of complicated shape, where minimum-weight design is not theoretically calculable. By dimensional argument he derives two coefficients, "stress resistance per unit volume" and "geometrical stiffness per unit volume", as measures of weight economy of design. Values of coefficients are measured on photoelastic model. Shape of model is altered by steps until coefficients reach maximum, which he states corresponds to minimum-weight solution. Two examples are given.

Reviewer believes derivation of "stress resistance per unit volume" is incorrect, and it is not always necessarily maximum for minimum-weight design. He finds, e.g., that author's method can lead to infinite cross section for minimum-weight design of simple rectangular beam, uniformly bent. Reviewer therefore advises caution in applying this method to complicated structures.

R. L. Woolley, Iraq

3881. **Simmons, J. C., A variable-air-gap inductance strain gauge,** *Mag. Concr. Res.* **6,** 19, 31–34, Mar. 1955.

This note describes a sensitive inductance strain gage designed for use on wet concrete test specimens. The gage is of the variable-air-gap type and depends on the strain of the test specimen causing a change in length of a small air gap in a magnetic circuit. An a-c bridge network and an amplifier enable the output signal to be measured by a d-c milliameter. The sensitivity of the gage can be varied over a wide range and, over the range of sensitivities tested, the calibration curve was sensibly linear. At maximum sensitivity, strains of the order of 2×10^{-7} could be detected.

From author's summary

3882. **Molyneux, W. G., and Ruddlesden, F., A technique for the measurement of pressure distribution on oscillating aerofoils, with results for a rectangular wing of aspect ratio 3.3,** *Aero. Res. Counc. Lond. curr. Pap.* 233, 10 pp. + 2 tables + 14 figs., 1956.

Details are given of a strain-gage pressure transducer that has been developed for measurements of pressure distribution on oscillating air-foils in low-speed wind tunnels. The transducer characteristics are shown to be well suited to oscillatory measurements, and, in particular, the transducer output can be measured directly on a sensitive galvanometer without the need for pre-amplification.

As an illustration of the use of the transducer, pressure measurements have been made in the R. A. E. 5-ft diam open-jet wind tunnel on a rectangular wing of aspect ratio 3.3 oscillating in modes of pitch and roll. Values for the aerodynamic derivatives have been obtained from the integrated pressure distributions, and are compared with those derived from over-all force measurements and with theoretical values. The measured values are in close agreement but there are some discrepancies with theory that are thought to be due to a wind-tunnel interference effect.

From authors' summary

3883. **Gerstle, K. H., and Clough, R. W., Model analysis of three-dimensional slab structures,** *Proc. Soc. exp. Stress Anal.* **13,** 2, 133–140, 1956.

Paper describes application of the powerful Beggs and Eney detormeter techniques to rectangular slab, bow girder slab, and cantilever stair slab to illustrate the general relevance of displacement model analysis methods to space structures. Apparatus is minor modification of that developed by Boyer [AMR **5,** Rev. 1365], who cited the same techniques, conclusions, and operational difficulties as present authors. Reviewer and Boyer [AMR **6,** Rev. 3053] elected to present influence surfaces for reactions as contour maps, considered superior to authors' isometric sketches.

G. P. Fisher, USA

Rods, Beams, Cables, Machine Elements

(See Also Revs. 3850, 3861, 3867, 3902, 3905, 3910, 3917, 3918, 3919, 3923, 3932, 3934, 3936, 3937, 4141, 4157)

3884. **Egervary, J., Application of matrix theory to chain bridges** (in Hungarian), *Magyar Tud. Akad. Mat. Fiz. Oszt. Közl.* **5,** 3, 301–313, 1955.

The structure considered consists of a series of rigid rods (the chain), of a horizontal elastic homogeneous prismatic beam (the bridge truss), and of equidistant rigid vertical members (the hangers) connecting the end points of the chain rods with the beam.

With the conventional assumptions, author gives method of computing deflections and moments for concentrated loads applied at the lower end of the hangers. Using the generalized Clapeyron equations, the problem is reduced to solving a system of inhomogeneous linear algebraic equations, i.e., to inverting a matrix the order of which is equal to the number of hangers. The matrix to be inverted is a linear function of a symmetric square continuant matrix with the properties: $A_{ij} = 2$, if $i = j$; $A_{ij} = -1$, if $i = j + 1$; all other elements being zero. The Fourier expansion of the solution is found by the canonical form of the afore-mentioned matrix.

Several insignificant misprints, few unconventional notations, and omission of some simple proofs make paper most enjoyable reading material for the well-prepared reader but at the same time exclude the majority of engineers from the audience. Useful reference (for suspension bridges where the chain is replaced by a cable) is "Mathematical methods in engineering" by von Karman and Biot, page 277.

V. G. Szebehely, USA

3885. **Szabo, J., Application of matrix methods to beams with elastic supports** (in Hungarian), *Mélyépítéstudományi Szemle* **6,** 2, 71–77, Feb. 1956.

Paper reviews elements of matrix theory and points out advantages in technical applications. Emphasized is the special continuant matrix mentioned in preceding review. An example is worked out in detail, computing the moments at the equidistant elastic support points of a prismatic beam.

V. G. Szebehely, USA

3886. **Ericksen, W. S., Bending and torsion of circular cylinder cantilever beams of cylindrically aeolotropic material,** *J. appl. Mech.* **23,** 2, 185–190, June 1956.

See AMR **9,** Rev. 1034.

3887. **Thorndike, A., Murphy, E. F., and Staros, A., Engineering applied to orthopedic bracing,** 122nd Meet., Amer. Assn. Advancement Sci., Atlanta, Ga. Dec. 26–30, 1955. 15 pp. + 3 tables + 9 figs.

Plates, Disks, Shells, Membranes

(See also Revs. 3859, 3861, 3871, 3873, 3912, 3913, 3922, 3957)

3888. **Frederick, D., On some problems in bending of thick circular plates on an elastic foundation,** *J. appl. Mech.* **23,** 2, 195–200, June 1956.

See AMR **9,** Rev. 2156.

3889. **Herrmann, J., and Ades, C. S., Experimental verification of circular plate bending theory,** *J. Franklin Inst.* **261,** 6, 631–635, June 1956.

Paper describes an experimental investigation involving two circular ring plates under a concentrated load along the inner edge and freely supported along the outer edge. The thickness of one plate was uniform, while that of the other decresed linearly from the bore to the rim. Strain measurements were taken at various radii to check the elementary elastic bending theory as applied to plates with both uniform and nonuniform profiles. Later, the disks were loaded to virtual failure as indicated by excessive deflections in order to determine their rupture factors. Finally, to evaluate the ultimate tensile strength needed for calculating those factors, a hardness check was made on just the uniform plate.

From authors' summary

3890. **Wu, C. S., Bending of rectangular plate with clamped edges,** *J. aero. Sci.* **23,** 6, 601–602 (Readers' Forum), June 1956.

3891. **Szelagowski, F., The problem of a couple acting on the periphery of a circular hole in an infinite plate,** *Bull. Acad. Polonaise Sci.* (IV) **3,** 3, 139–144, 1955.

The problem of the computation of stresses in an infinite plate with a circular hole under the action of a couple of forces acting on the periphery of the hole is examined on the basis of the principle of superposition as composed of two different problems; the first concerning normal forces and the second, tangential forces. The stress distribution corresponding to the first problem, involving normal forces, has already been found by the author in a previous paper. The present paper restates these previously found results and then treats the second problem. The latter is solved using Poisson's integral. Finally, the results of the two problems are superposed to give the complete solution.

R. L. Bisplinghoff, USA

3892. **Radkowski, P. P., Stresses in a plate containing a ring of circular holes and a central circular hole,** *Proc. second U.S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 277–282.

The plate is in a state of generalized plane stress and subjected to constant radial tension at infinity. The stress function is determined by constructing and then combining two series of periodic biharmonic functions. The first series relates to the infinite plate containing one circular hole; the second series relates to the ring of holes.

From author's summary by F.R.N. Nabarro, South Africa

3893. **Czerny-Wein, F., Tables for uniformly fully loaded rectangular plates** (in German), *Bautech.-Arch.* no. 11, 33–87, May 1955.

Detailed tables and graphs for displacements, bending moments, and shearing stresses are given for uniformly loaded rectangular plates with the following edge conditions: (1) All edges simply supported or clamped; (2) one or more edges simply supported and the rest clamped.

Designers will find the tables very useful.

B. R. Seth, India

3894. **Riparbelli, C., General bending of a plate into a developable surface, Note III,** *Cornell aero. Lab. Rep.* OSR-TN-56-31, 12 pp. + 4 figs., Dec. 1955.

The internal forces acting on a straight line section (generator) of a developable surface are assumed to consist of a moment about the line and shears perpendicular to the line. The relation between the moment and shear are determined by statics. In order to completely determine the straight lines (termed lines of maximum curvature by author), it is necessary to minimize the energy integral with respect to two unknown function of these lines. The minimization is simplified by determining the relation between the deformation energy of a plate subjected to pure bending and one subjected to combined bending and shear, as is the case here. The relation is given in the form of curves.

K. Arnstein, USA

3895. **Riparbelli, C., Edge curling of a plate undergoing oblique bending, Note IV,** *Cornell aero. Lab. Rep.* OSR-TN-56-32, 15 pp. + 4 figs., Dec. 1955.

By oblique bending, author apparently means twisting of thin rectangular plates. The deformed plate describes approximately a developable surface except at the edges, where the generators are bent. This edge effect is accounted for by considering the given plate as an internal region of a larger plate, and then applying the computed shears and moments to the deformed plate with opposite sign.

K. Arnstein, USA

3896. **Riparbelli, C., Geometry of the lines of maximum curvature in bending of thin plates, Note V,** *Cornell aero. Lab. Rep.* OSR-TN-56-33, 9 pp. + 3 figs., Dec. 1955.

On the assumption that the bending deformation of a plate is caused by the moment components parallel to the lines of maximum curvature, the curvature of such lines determines the deflections of a plate under a certain loading. This can be done only when the lines of maximum curvature are parallel. When special geometric considerations, such as symmetry, are not available, a family of parallel straight lines is suggested for the representations of the lines of maximum curvature. The orientation of these lines can be determined by the principle of minimum total potential energy. The curvature of these lines then can be determined by the edge conditions. No application of the method is given, because this is an addendum to the concept presented by the same author in Note I of this series.

K. Arnstein, USA

3897. **Seide, P., On the torsion of rectangular sandwich plates,** *J. appl. Mech.* **23,** 2, 191–194, June 1956.

See AMR **9,** Rev. 1414.

3898. **Morley, L. S. D., Determination of the stress distribution in reinforced monocoque structures. Part I. A theory of flat-sided structures,** *Aero. Res. Counc. Lond. Rep. Mem.* 2879, 23 pp., 1955.

Paper presents a theory for the estimation of the stress distribution in the vicinity of discontinuities, such as an abrupt change of a section or a concentrated load which might occur in a flat panel made up of sheet and stringers. The author, in his mathematical analysis assumes: (1) The stringers to be uniformly distributed; (2) the stringers carry only direct load; (3) the thin sheet can resist only shear loads, account being taken of its contributory direct stiffnesses by corresponding increases in the stiffener cross section; (4) the rib flanges can resist only direct load; (5) buckling will not occur.

The analysis develops the fundamental equations which govern the stresses, strains, and displacements in the sheet stringer combination and the rib flanges separately. Using these fundamental

equations, differential equations of compatibility are formed. The solution of the differential equations yields the stress distribution in the panel.

Author applies his theory to the problem of a three-bay flat structure with a rectangular cut-out in the center bay and subjected to uniform tension. The calculated results are compared to test results.

Further papers are planned which will apply the equations developed in this paper to solutions of specific problems. The results will be verified by tests. K. Arnstein, USA

3899. **Durelli, A. J., and Barriage, J. B., Calculating stresses in pressurized square tubing having circular central holes,** *Mach. Design* **28,** 5, 115–117, Mar. 1956.

An experimental investigation of the stress field in a square plate containing a circular hole with a uniform radial pressure acting in this circular hole. If D is the hole diameter, and a the side dimension of the square, the data cover values of the D/a ratio of from 0.3 to 0.85 and determine the location and magnitudes of the maximum shear and normal stresses in the material. No theoretical solution has been attempted, but the experimental data are sufficiently complete for design purposes. Whether or not the data could be used without modification for a long cylinder with the same cross section may be open to question, since the effect of axial restraint on displacements in that direction may modify the results; however, the curves should at least give first-order values for a square tube with a circular hole under pressure.

E. F. Sechler, USA

3900. **Kafka, P. G., and Dunn, M. B., Stiffness of curved circular tubes with internal pressure,** *J. appl. Mech.* **23,** 2, 247–254, June 1956.

See AMR **9,** Rev. 1769.

3901. **Thomson, A. S. T., Scott, A. W., Moir, C. M., Shrink-fit investigations on simple rings and on full-scale crankshaft webs,** *Proc. Instn. mech. Engrs.* **168,** 32, 797–830, 1954.

Authors present thorough experimental results, examining surface conditions and fit allowances on small-scale tests. Surface finish and lubrication conditions are systematically varied. Fit allowance varies from 0.5 to 4.0 mils per inch diam. Theories used for comparison are: elastic (Lamé), plastic (Allen and Sopwith AMR **4,** Rev. 3181). Good agreement is shown for elastic state but not the plastic, which is not surprising. The latter theory assumes no work-hardening and flow according to constant-shear-stress theory. Employing Mises-Hencky criterion and taking account of work-hardening, better agreement would be obtained.

On full-scale crankshaft webs, fit allowance varies from 1.0–2.0 mils per inch diam. Variation in radial and circumferential stresses are examined. Diagrams are presented.

Some materials details are omitted but paper is stimulating.

P. W. Whitton, Australia

3902. **Soule, J. W., Tensor-flexibility analysis of pipe-supporting systems,** *J. appl. Mech.* **23,** 2, 181–184, June 1956.

See AMR **9,** Rev. 1767.

3903. **Clark, R. A., and Reissner, E., On axially symmetric bending of nearly cylindrical shells of revolution** *J. appl. Mech.* 23, 1, 59–67, Mar. 1956.

A "nearly cylindrical" shell is defined as a corrugated shell in which the amplitude of the corrugation is small compared to the mean radius and the slope of the corrugation is small compared to unity. Appropriate differential equations are obtained from shell theory and solutions are obtained by a perturbation procedure involving an expansion in powers of two parameters which are small for "nearly cylindrical" shells. The results are compared with previous theory and a single experimental point and show good agreement. Method is suitable for any shape of corrugation; numerical results are given for sinusoidal, parabolic, and saw-tooth corrugations. N. C. Dahl, USA

3904. **Gibson, J. E., The application of matrices to the design of long cylindrical shell roofs. Parts I, II, III,** *Civ. Engng. Lond.* **50,** 590, 863–865, Aug. 1955; **50,** 591, 992–994, Sept. 1955; **50,** 592, 1121–1123, Oct. 1955.

This paper is the immediate continuation of a former one [title source, Oct '53–Jan '54] apparently not discussed in these Reviews. There, author compared simplified solutions of the shell problem solved numerically by the slide rule with more complete ones obtained with the help of calculating machines. Here author's aim is threefold: to give the former calculations (for the "long" roof—those for the "short" roof being rather simple) a more concise form by the use of matrix symbolism, to find checks for the details of the rather lengthy calculation, and to complete the analysis by including antimetric load and more general edge conditions.

Designer interested in such problems may also be referred to author's book "Design of cylindrical shell roofs," London 1954.

K. Marguerre, Germany

3905. **Horvay, G., and Clausen, I. M., Membrane and bending analysis of axisymmetrically loaded axisymmetrical shells,** *J. appl. Mech.* **22,** 1, 25–30, Mar. 1955.

Authors present membrane formulas for the frequently occurring cases of conical and spherical shells loaded by weight, gas pressure, and liquid pressure. Shell-bending formulas, based on the Geckeler approximation, are also listed. Paper serves as a supplement to texts on stress analysis formulas and an aid in design of shell structures. Authors present tabulation of the moment and shear attenuating factors as a function of the nondimensional distance from the shell edge. Similar tables in authors' reference (3) require less calculation and reduce the possibility of numerical error in calculating the argument of the trigonometric functions.

L. W. Smith, USA

3906. **Chiboeff, V. G., Minimum weight of cylindrical tanks subjected to internal pressure and with elliptical or hemispheric bottom** (in French), *Rev. Inst. fr. Petrole et Ann. Comb. liq.* **10,** 10, 1284–1294, Oct. 1955.

Author determines the weight of cylindrical tanks with elliptical or hemispheric bottom. Using an approximate formula, he then establishes the optimum ratio of length to diameter. This optimum ratio corresponds to the minimum weight of the tank, for fixed values of design pressure and capacity.

G. Supino, Italy

Buckling Problems

(See also Revs. 3859, 3861, 3936)

3907. **Pearson, C. E., General theory of elastic stability,** *Quart. appl. Math.* **14,** 2, 133–144, July 1956.

Using the Murnaghan finite deformation theory ["Finite deformations of an elastic solid;" Wiley, 1951] for elastic solids, author derives a criterion for instability of equilibrium of body in terms of internal energy change and virtual work of external loads. The difference between this criterion and that based on the usual assumption of adjacent equilibrium states is carefully discussed. Starting with the general principle, author makes customary engineering approximations and shows that the familiar Euler formula results for buckling of a slender column under thrust. Other cases examined are flexural-torsional buckling, in which the results of Goodier [title source, 1944] are duplicated, and buckling of a cylindrical shell under external uniform loading. In the latter case, the necessity for specifying the manner in which the load (in magnitude and direction) changes with the deformation of the shell is clearly presented. H. J. Plass, Jr., USA

3908. **Ziegler, H., On the concept of elastic stability,** pp. 351–413, Advances in Applied Mechanics, Vol. IV. (Dryden, H. L., von

Karman, T., and Kuenti, G., editors), New York, Academic Press, Inc., 1956.

This critical analysis of methods in use for study of the stability of elastic equilibrium is confined to systems of the holonomic and the scleronomic types with differential equations that can be linearized. Attention is drawn to the fact (pointed out in some of author's earlier publications) that the concept of stability itself on which the classic approach is founded may lead to erroneous results. His main examples are the buckling of a column and the critical angular velocities of a rotating shaft.

Author distinguishes three static methods: (1) The "imperfection method" (what is the value of the load for which the deflections of the system increase beyond any limit?); (2) the equilibrium method (what are the values of the load for which the system admits nontrivial equilibrium configurations?); (3) the energy method (what is the value for which the potential energy ceases to be positive definite?), and one kinetic method: (4) the vibration method (what is the value of the load for which the most general motion of the system ceases to be bounded?) Examples are given where the methods give different results. [There is one on p. 361 showing that, according to author's way of reasoning, the kinetic method fails, while correct results are supplied by (1) and (2)]. Therefore, author gives a classification of mechanical systems in order to establish limits within which the methods may be safely applied. The classification consists of two conservative types (nongyroscopic and gyroscopic systems) and three nonconservative ones (dissipative, circulatory, and unstationary systems). Only linear systems of a finite number of degrees of freedom are taken into consideration.

A definition of stability is given and a new concept is introduced, latent instability, which plays an important part in what follows. Several theorems are given, such as problems of the circulatory type cannot be solved by static methods. The theoretical results are applied to columns and shafts. A column loaded tangentially is shown to be circulatory. Buckling by torsion is considered and Greenhill's formula verified. Critical speeds of shafts give other applications, the case of gyroscopic moments and that of unequal flexural rigidities being treated. Author's general conclusion is that the correct approach to the solution of a stability problem essentially depends on the nature of the problem. The kinetic approach is frequently questionable if physical imperfections are neglected. O. Bottema, Holland

3909. Procter, A. N., Buckling under complex loading, Part I, II, *Engineer, Lond.* **201,** 5237, 629-632, June 1956; **201,** 5238, 667-669, June 1956.

Formulas are developed for elastic buckling stresses in structural members subjected to various combinations of axial load, transverse load, and torque applied at the ends. It is assumed that the cross section is doubly-symmetrical, uniform throughout the length, and of such proportions that the effects of restraint of warping can be neglected. Strain energy method of calculation is used. The deflected shape of the axis is assumed to be represented by a single cosine term.

Author discusses results of tests on centrally loaded and eccentrically loaded columns of various materials that have been reported by others. Reviewer believes that several of the author's statements are misleading. For example, the sentence, "By deflecting slightly the axially loaded 'flexible' strut avoids a combination of crushing and bending stresses and is thus able to carry more load than otherwise..." implies that a slightly crooked column is stronger than a straight one. J. W. Clark, USA

3910. Bőröcz, I., Influence of inaccuracies of execution upon the stability of beams of rectangular cross section, suspended at both ends (in Hungarian), *Mély êpîtêstudomānyi Szemle* **6,** 3, 105-115, Mar. 1956.

In the erection of prefabricated reinforced-concrete structures, beams of the shape of planks on edge are generally hoisted into position, suspended at both ends. It is known that such beams are liable to buckle sidewise [AMR **7,** Rev. 3527; AMR **8,** Rev. 65; AMR **9,** Rev. 715]. New, however, is the presentation of the effect of faulty execution on safety. The paper deals in detail with the influence of the following defects of execution: (a) the axis of the beam has initial curvature; (b) cross sections of the beam are already originally twisted in relation to one another; (c) the beam is not suspended precisely at the prescribed points. Planning by the limit-design method is presented with consideration of these possibilities of faulty execution. A numerical example closes the paper. P. Csonka, Hungary

3911. Negoro, S., On the elastic failure and the buckling of a column under eccentric loads, *Rep. Res. Inst. appl. Mech. Kyushu Univ.* **4,** 14, 41-56, Oct. 1955.

Author solves title problem for conventional case where both eccentricities lie in plane through one principal axis and also for case when they are arbitrary. Elastic failure is considered to take place when maximum stress equals yield stress. Author proposes to extend method to plastic range, but appears to be unaware of extensive literature available on subject. (See for example, Hoff, AMR **7,** Rev. 2444.] P. C. Dunne, England

3912. Klein, B., Buckling of simply supported plates tapered in planform, *J. appl. Mech.* **23,** 2, 207-213, June 1956.

See AMR **9,** Rev. 2155.

3913. DiMaggio, F. L., Effect of an acoustic medium on the dynamic buckling of plates, *J. appl. Mech.* **23,** 2, 201-206, June 1956.

See AMR **9,** Rev. 1424.

3914. Schmitt, A. F., A method of stepwise integration in problems of impact buckling, *J. appl. Mech.* **23,** 2, 291-294, June 1956.

See AMR **9,** Rev. 1779.

Book—3915. Baker, J. F., Horne, M. R., and Heyman, J., The steel skelton. Vol. II, Plastic behavior and design, New York, London, Cambridge University Press, 1956, × + 408 pp. $12.

Prof. Baker's first volume on "Elastic behaviour and design" [AMR **7,** Rev. 3190] has been followed by a companion volume on "Plastic behavior and design", which contains an authoritative statement of the principles and extensive experiments carried out mainly by the senior author and his team during the past 20 years, first at Bristol and later at Cambridge, England. The detailed account of this important engineering research and its development to a comprehensive design method will interest all engineers concerned with steel structures.

The book opens with a brief review of earlier work on the subject and proceeds to discuss the simple plastic theory of bending and its application to single and continuous beams. Experimental evidence shows that this theory gives predictions of the collapse load of such beams with sufficient accuracy for practical purposes, including cases of sinking supports and variable end connections, provided they can resist the full plastic moment. In the following chapters, the general concept of plastic hinges is applied to rectangular and pitched-roof portal frames, and the theoretical modes of failure and collapse loads are verified by tests on models and full-scale frames. The results of the full-scale tests, which included portals on pad footings, and the detailed designs should give structural engineers full confidence, since steel portal frames behave at collapse as the theory indicates.

Before treating the more complicated structures and the problems concerned with multistory building frames, authors discuss the fundamental theorems of Greenberg and Prager and others and proceed to apply them in their treatment of general methods of analysis. The most general of these methods are the analysis by combination of elementary mechanisms (upper limit of collapse load) and the plastic moment-distribution method (lower limit of collapse load), and they are well illustrated by examples. Subsequently, minimum weight design, shakedown, and the effects of shear and

lateral instability of beams are considered. One of the most difficult problems in frame structures designed by the ultimate load method is the plastic behavior of columns in multistory buildings, and several important chapters are devoted to this subject. Early tests on single-curvature and double-curvature bending of columns are reviewed, and an elastic-plastic theory of columns bent in one plane is developed and compared with test results. Finally, an analysis of bending of columns about two axes of symmetry is made and the results are represented on charts, which enable designers to check the adequacy of a column section once the axial load and end moments determined from the most critical loading condition are known.

The concluding chapters of the book deal with the load factor and practical experience in the application of the plastic method of design. The success obtained shows that the collaboration between research workers and practicing engineers has produced a rational design method for many steel structures. This achievement is, to a large degree, due to authors' pioneering work collected in the present book, which, together with its companion volume, contains, in the reviewer's opinion, one of the most outstanding contributions to the knowledge of the actual behavior of steel building frames. G. G. Meyerhof, Canada

Structures

(See also Revs. 3880, 3883, 3886, 3891, 3897, 3898, 3904, 3906, 3910, 3934, 3943, 3955, 3963, 3979, 4115, 4120, 4121, 4122, 4123, 4141, 4156)

3916. **Ringo, B. C., Limit design for buildings,** *Proc. Amer. Soc. civ. Engrs.* **82,** ST 3 (*J. Struct. Div.*), Pap. 986, 15 pp., May 1956.

Article compares application of limit-design method to elastic-design methods as used with structural steel. The method of limit design is applied to typical building construction, but is restricted to the beam and girder floor framing system. The comparative designs are carried out for a total of six design cases of varying live load and panel sizes. A summation of structural steel weights for typical panels and for multiple floors composed of these panels is then presented to point out the relative steel requirements for the limit-design procedure and for the elastic-design procedures. In conjunction with the basic objective of comparative weights, this paper points out the relative simplicity of the limit-design procedure.

Also, a brief analysis of the limit designs will show the existing stresses and deflections under design loads, as well as their remaining capacities. As an aid to the limit or plastic-design procedure, a tabulation of plastic section properties for a series of wide-flange beams is included.

Paper is intended to present some of the advantages and inherent characteristics of a simplified application of the method of limit design as applied to typical structural steel framing for multi story buildings. From author's summary

3917. **Falk, S., Calculation of a cantilever beam according to the reduction method** (in German), *Ing.-Arch.* **24,** 3, 216–232, 1956.

The reduction method has been used for quite some time in applied physics and mechanics. Author develops this method for statically indeterminate structures. Formulas in the form of matrixes are given which could be applied efficiently by computers. Three numerical examples show that the method is superior to the conventional elimination method, as far as the computing effort and the scope are concerned.

C. B. Ludwig, USA

3918. **Yoshimura, T., Analysis of rigid frames by balancing member series,** *Mem. Fac. Engng. Kumamoto Univ.* **2,** 1, 40 pp. + 9 tables, 1955.

The bending moments in continuous beams and frames are found by a short cut extension to the Hardy-Cross method because the final balanced moments are obtained in one step, as compared to the successive approximations otherwise necessary. Paper shows that the successive approximations can be eliminated because moments carried over to a given joint arrange themselves in a geometric series, hence their sum, and therefore the final balanced moment, is known. Table of sums of carry-over coefficients for beams of various numbers of spans up to seven, along with illustrative problems, is also given. D. W. Breuer, USA

3919. **Yoshimura, T., and Murakami, T., Balancing member series by parts,** *Mem. Fac. Engng. Kumamoto Univ.* **3,** 1, 10 pp., 1955.

The preceding review shows how carry-over moments in multiple-span beams can be obtained without the use of successive approximations. The first author's initial paper tabulated the sum of the carry-over moment coefficients for beams up to seven-span length.

This paper deals with using these tabulated results on any continuous beam or frame with span number greater than three or four by breaking the structure down into continuous beams of only three or four spans. These continuous beams are termed "member series" by the authors. Two methods are presented, one by each author, and example problems are shown to illustrate each method.

The procedures developed in these two papers can greatly facilitate the solution of complicated structures problems consisting of multiple-span and multiple-level beams, frames, and bents.

D. W. Breuer, USA

3920. **Goodey, W. J., Notes on a general method of treatment of structural discontinuities,** *J. roy. aero. Soc.* **59,** 538, 690–693, Oct. 1955.

3921. **Szilard, R., Jr., Strengthening steel structures by means of prestressing,** *Engng. J., Montreal,* **38,** 10, 1379–1381, Oct. 1955.

Author analyzes the strengthening of steel structures by the addition of cables which, being tensioned, modify stress state favorably. A bar in traction and a simply supported beam are studied as examples.

It is reviewer's opinion that the word "prestressing" is not correctly used in this case. A. J. Bignoli, Argentina

3922. **Wang, C. T., Ross, A. L., Reiss, E. L., An investigation of a method for prestressing flat plates to increase their buckling strength,** *WADC TR* 54-8, 190 pp., Feb. 1954.

Investigation deals with determining the extent to which prestressing flat plates (by cold rolling the plate into a cylindrical form and then opening the curved plate by riveting it to a planar frame) raises the buckling load in a direction perpendicular to the direction of rolling. 24S-T sheet material was used. The study, which was restricted to the case of all four sides being clamped, indicates the importance of the in-plane stresses, rather than favorable deformations, as the primary factor in raising the buckling loads.

The stresses produced by the above procedure were measured and their effect on buckling strength was calculated by the Raleigh-Ritz method. The analysis showed that small stresses (1000 psi maximum) could raise the buckling load 50% or better and, in some cases, even change the mode of buckling from antisymmetrical to symmetrical modes. Actual tests showed that the buckling load in some cases was raised over 100%, with an average increase of 38% for all tests.

A theoretical evaluation of the in-plane stresses and their effect on the buckling load was also carried out, using the nonlinear plate equations of von Karman. The increase in buckling load predicted by this analysis was considerably less than that actually measured. This, at least in part, was due to the inaccuracy of the assumptions. M. M. Lemcoe, USA

3923. **Prentis, J. M., The effect of varying the pretension and area of reinforcement on the ultimate bending strength of concrete beams,** *Mag. Concr. Res.* **7,** 21, 143–150, Nov. 1955.

Tests on eighteen pretensioned concrete beams are reported. The beams were cast in groups of three, the members of any one group differing only in the amount of pretension. The percentage of reinforcement varied between groups but was always sufficient to preclude failure by steel rupture.

Two series of tests were made. In the first, three groups of three beams were made and tested by the author who used an extreme range of pretension. The second series was made and tested independently by Mr. F. C. H. Nisbet, who repeated the author's tests in respect of beam dimensions and percentage of reinforcement, but who used rather higher concrete strengths and a more practical range of pretension.

The beams were all 4 in. deep with an effective depth of $2\frac{1}{2}$ in. Two widths were used: $1\frac{1}{8}$ in. and $1\frac{1}{2}$ in. The length between supports was 45 in., and under four-point loading the central 15 in. was subjected to a uniform bending moment.

Both the area of steel and the degree of pretension were found to have an appreciable effect on the bending strength of the beams.

From author's summary

3924. Hendry, A. W., and Jaeger, L. G., The load distribution in highway bridge decks, *Proc. Amer. Soc. civ. Engrs.* **82,** ST 4 (*J. Struct. Div.*), Pap. 1023, 48 pp., July 1956.

The general method for calculating the distribution of longitudinal moments, deflections, etc., in bridge decks is based on the assumption that the transverse system with different torsional rigidity is replaced by uniform medium of the same total flexural rigidity. Under this assumption it is relatively easy to derive distribution coefficients for the harmonic components of applied loads, free bending moment diagram, or free deflection curve. It is shown that a bridge of a large number of longitudinal beams can be analyzed with sufficient accuracy by consideration of five longitudinals. This method is also applied to slabs. Comparisons between theoretical and experimental results are made for steel and concrete tee-beams, steel beams and jack arches, beams and slabs, tiller joist decks, and reinforced-concrete slabs. Special reference is made to previous research by S. Timoshenko, N. M. Newmark, C. Massonet, and others, and also to authors' papers, some of them not published yet.

J. J. Polivka, USA

3925. Jaeger, L. G., and Hendry, A. W., The analysis of interconnected bridge girders by the distribution of harmonics, *Struct. Engr.* **34,** 7, 241–266, July 1956.

Paper describes a new method for the analysis of simply supported interconnected bridge girders. The floor slab and cross beams of the bridge are replaced by a uniform medium without torsional rigidity, of total moment of inertia equal to that of the actual transverse system. A differential equation including terms due to rotation and twist is written down for each longitudinal. These equations are solved by harmonic analysis, and distribution coefficients are derived giving the amplitudes of the harmonics of the bending moment or deflection curve for each girder. The theory presented allows for any degree of torsional stiffness of the longitudinals and, in the most general case, the distribution coefficients are functions of three dimensionless parameters; the first is a measure of the transverse stiffness of the bridge, the second of the torsional stiffness, and the third expresses the ratio of the inertias of the outer to the inner longitudinals.

For design purposes, the cases of zero and infinite torsional stiffness are of greatest interest, but intermediate cases can be obtained by use of an interpolation formula. An alternative and very simple method is developed for zero torsional stiffness and the calculation of transverse moments is also dealt with. Numerical applications of the method to the design of a steel and a concrete bridge are given. Finally, authors compare their method, as well as the theory replacing the bridge grid by an anisotropic plate and due to Y. Guyon and this reviewer, with experimental results obtained by Morice and Little. The agreement is excellent and it is shown that both theories give quite the same results.

The method presented seems to be one of the few which allows practical application of the effect of the torsional rigidity of the bridge girders. Its principal defect seems to be the impossibility of taking into account the torsional rigidity of the cross beams; also, the theory has been developed analytically for bridges with three or four longitudinals only, and no numerical tables are given.

As the merits and drawbacks of the theory of Guyon and this reviewer are discussed in the introduction of the paper, it may be emphasized that, in a recent paper [*Ann. Trav. publics Belg.* October 1954], the Guyon-Massonnet theory has been extended to the case in which the outer girders are of a different section and full numerical tables are given allowing the speedy calculation of practical problems.

C. Massonnet, Belgium

3926. Edgerton, R. C., and Beecroft, G. W., Dynamic stresses in continuous plate girder bridges, *Proc. Amer. Soc. civ. Engrs.* **82,** ST 3 (*J. Struct. Div.*), Pap. 973, 27 pp., May 1956.

This paper describes and presents results of tests conducted by the Bureau of Public Roads and the Oregon State Highway Department on two three-span continuous plate girder bridges in Oregon. Measurements include deflection of the girders and strain in the girders, stringers, and floor beams under test vehicles approximating the AASHO H20-S16-44 truck at speeds of 5 to 45 mph. The test records provide values of frequency of vibration, amplitude of vibration, and amplitude of strain oscillation. Test data are presented in curves showing variation of stress and deflection with speed. Comparisons are made with stresses and deflections calculated according to existing specifications. Measured stresses in general are found to be lower than calculated values. Analytical work pertaining to theoretical frequencies and amplitudes of vibration is summarized. A good correlation was obtained between measured and computed frequencies of vibration. Calculations relative to the amplitude of vibration do not provide a correlation with the measured values; however, they strongly suggest that a substantial part of the vibration can be attributed to deck roughness. Measured strains are used to determine the degree of fixity of the ends of the floor beams, and measured deflections are used to evaluate the torsional rigidity of the structures.

From authors' summary

3927. Hunt, P. M., The electronic digital computer in aircraft structural analysis. Part I. A description of a matrix interpretive scheme and its application to a particular example, *Aircr. Engng.* **28,** 325, 70–76, Mar. 1956.

Paper describes an interpretive scheme which enables an electronic digital computer to deal with matrixes and matrix instructions. Author applies the method in determining the instruction program for solving a particular redundant structure.

S. F. Borg, USA

3928. Hunt, P. M., The electronic digital computer in aircraft structural analysis. Part II. The use of present and programme parameters with the matrix interpretive scheme and their application to general purpose programmes for the force method of analysis, *Aircr. Engng.* **28,** 326, 111–118, Apr. 1956.

Author considers special parameters for use in the method described in part I [see preceding review] and the use of these in general-purpose programs for redundant structural analyses. A discussion of matrix partitioning is also included.

S. F. Borg, USA

3929. Hunt, P. M., The electronic digital computer in aircraft structural analysis. Part III. General purpose programmes for the force and displacement methods in large structures including the use of magnetic tape storage, *Aircr. Engng.* **28,** 327, 155–164, May 1956.

Paper develops the general-purpose program (including all the methods described in parts I and II; see two preceding reviews) for the digital computer solutions of the statically indeterminate structure. Detailed programs are given and an outline is given of the

procedure to be followed for the analysis of a delta wing.

Author discusses the use of magnetic tape storage.

S. F. Borg, USA

3930. Ebner, H., The status of research on strength problems concerning light construction, especially aircraft construction (in German), *Z. Flugwiss.* **4**, 3/4, 108–121, Mar./Apr. 1956.

A review is given of the status of research on a series of strength problems concerning light construction, especially aircraft construction. The report comprises the development of investigation methods on the state of stress in stiffened sheet-metal constructions with or without cutouts, the research results on fatigue stress of light construction elements, and on thermal stress of aircraft construction elements, a problem which has become most important with regard to the high speeds of modern aircraft and guided missiles. A summary is given on several new problems related to the stability of light construction elements, such as skin covered bars, cylindrical shells, and stiffened spherical shells. Experimental methods for the investigation of theoretically complicated strength problems are also discussed.

From author's summary

3931. Bateman, E. H., An engineering approach to some structural problems arising from kinetic heating, *J. roy. aero. Soc.* **60**, 546, 402–407, June 1956.

3932. Johnson, W., The effect of curvature on the centre of shear, *J. roy. aero. Soc.* **59**, 536, 562–565 (Technical Notes), Aug. 1955.

3933. Nonweiler, T., The design of wing sections. A survey of existing knowledge on aerofoil design for different conditions, *Aircr. Engng.* **28**, 329, 216–227, July 1956.

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 3858, 3864, 3869, 3879, 3901, 3915, 3956, 3957, 4113)

Book—3934. Phillips, A., Introduction to plasticity, New York, The Ronald Press Company, 1956, ix + 230 pp. $7.

Content of book is well expressed in opening sentence of introduction: "This is an introductory text on the subject of metal plasticity, written from the point of view of the stress analyst."

First six chapters are concerned with one-dimensional stress problems: trusses, beams, and frames. Chapter 1 introduces stress-strain diagram for simple tension beyond elastic limit. Next, simple plastic collapse load concepts are reviewed. Chapter 3 gives details for finding moment-curvature relations for arbitrary beam sections and arbitrary stress-strain curves. Next three chapters are concerned with the influence of axial forces in bending, plastic displacements, and curved bars.

Last three chapters take up combined stresses. Chapter 7, stress-strain relations, is based wholly on the Mises yield conditions and Prandtl-Reuss stress-strain laws with no indication that there might be other approaches. Chapter 8 finds collapse loads of spheres, cylinders, flat annular rings, wire drawing, and problems in tension and plane strain. The final chapter considers some of the same problems for strain-hardening plasticity.

Treatment throughout is elementary and requires no background beyond strength of materials. Mathematics is presented in full detail. Many results are presented in comprehensive tables which may be of value to analysts and designers. Further simplicity is achieved by omitting discussion of conflicting theories in plasticity. This may lull readers into false sense of validity of some results. Stress analysts and teachers using this for course work must also refer to other texts for full appreciation of limitations of book. But as an "introduction" it is readable, should be of immediate value because of its detail, and should lead to more engineers becoming aware of the subject of plasticity.

P. G. Hodge, Jr., USA

3935. Prager, W., A new method of analyzing stresses and strains in work-hardening plastic solids, ASME, Nat. Conf. of appl. Mech. Div., Urbana, Ill., June 1956. Pap. 56-APM-34, 4 pp.

Total stress-strain laws in plasticity would have the advantage of being much easier to handle than incremental laws, but, in general, total laws are physically unsound. Author offers a compromise between these two types of stress-strain laws by using a segmentwise linear yield condition in conjunction with the appropriate flow rule, applied to a rigid, work-hardening material in plane stress. Thus he uses the mathematical advantages of the total laws without sacrificing the essential features of the incremental laws. The mechanical behavior of the plastic solids is illustrated by rather simple kinematic models. The bending of a simply supported circular plate by a uniformly distributed transverse load is treated as an example. F. Chmelka, Austria

3936. Lin, T. H., Creep stresses and deflections of columns, *J. appl. Mech.* **23**, 2, 214–218, June 1956.

See AMR 9, Rev. 1442.

3937. Conroy, M. F., Plastic deformation of semi-infinite beams subject to transverse impact loading at the free end, *J. appl. Mech.* **23**, 2, 239–243, June 1956.

See AMR 9, Rev. 1383.

3938. Tobolsky, A. V., Stress relaxation studies of the viscoelastic properties of polymers, *J. appl. Phys.* **27**, 7, 673–685, July 1956.

Author reviews studies on the viscoelastic properties of polymers undertaken in his laboratory. Discussion is divided into four parts. (1) The phenomena of chemical stress relaxation in the cross-linked rubbers at elevated temperatures can be attributed to the chemical reactions, such as chain scission by oxidative cleavage or reorganization of the network structure by ionic interchanges. Experimental verifications are made against this concept. (2) From the stress relaxation data of amorphous polymers at various temperature ranges, four regions of viscoelastic behavior are defined, and the master curve of $\log E(t)$ versus $\log (t)$ can be constructed by means of the time-temperature superposition principle. Parameters which characterize the master curve are defined, and the relation between these parameters and molecular structure is discussed. (3) The relaxation experiment of polycrystalline polymer (Kel-F) is described, and the relation between stress decay at constant extension and crystallization of polymer is discussed. (4) Stress relaxation studies are under way with wool, gelatin, and collagen. H. Mii, Japan

3939. Mooney, M., Theory of the non-Newtonian rheology of raw rubbers consisting of supermolecular rheological units, *J. appl. Phys.* **27**, 7, 691–696, July 1956.

A theory of the viscosity of raw rubbers is developed on the postulate that raw rubbers, when subjected to continuous shear in the non-Newtonian region of flow, consist of microscopic rheological units which are semipermanent aggregates of many rubber molecules [M. Mooney and W. E. Wolstenholme, title source, **25**, p. 1098, 1954]. The theory treats these units as tacky, elastic solids, whose relative movement and slippage constitute the macroscopic flow of the rubber. The resulting theoretical flow curve differs little from that given by H. M. Smallwood [title source, **8**, p. 505, 1937], but the interpretation of the parameters is radically different. The new theory leads to expressions for the number of temporary point attachments per unit area between two touching rheological units and for the mean life of these attachments. Agreement with experimental data is slightly better than in Smallwood.

Two types of observed deviation from the theory can be interpreted as due to two plausible phenomena not included in the mathematical treatment.

From author's summary by H. Mii, Japan

3940. Mandel, J., On viscoelastic bodies in the linear region (in French), *C. R. Acad. Sci. Paris* **242**, 2803–2805, June 1956.

Author writes down some conditions satisfied by a vibrating viscoelastic body. Reviewer feels that a two-page note on such a topic is inadequate and that author should develop his ideas further and relate them to previous work.

D. R. Bland, England

3941. Yamamoto, M., The viscoelastic properties of network structure, I. General formalism, *J. phys. Soc. Japan* **11**, 4, 413–421, Apr. 1956.

In this paper, author extends and applies the Green and Tobolsky treatment of the relaxation of a rubberlike network model to cases with more than one relaxation time and where probability of breakage of chains is not constant. It is stated that model corresponds phenomenologically to the "generalized" Maxwell model and that the assumption concerning the dependence of the probability of breakage of chains will make it possible for model to accommodate anomalous rheological properties of the systems, this latter aspect to be shown in a future paper.

Except for assumption concerning variation in the probability of breakage of chains, approach is similar to that of Lodge [Proc. sec. Inter. Congress on Rheology, New York, Academic Press, Inc., 1954].

Y.-H. Pao, USA

Failure, Mechanics of Solid State

(See also Revs. 3931, 4102, 4113)

3942. Weibull, W., Effect of crack length and stress amplitude on growth of fatigue cracks, *Flygtekn. Försöksanst. Medd.* 65, 44 pp., 1956.

A study of the rate of growth of fatigue cracks. Earlier observations on this important phenomenon have been obscured by the fact that in previous tests the stress amplitude increases as the crack propagates. In this investigation 24S-T and 75S-T aluminum specimens were run with carefully controlled constant stress amplitude. The results suggest that the crack growth is independent of crack size but highly dependent on small variations in the stress amplitude. It is significant that the growth rate is much higher for the 75S-T, in spite of its higher static strength. This investigation, with its mature attention to statistical interpretation, is a worthwhile addition to the literature on the subject.

C. R. Soderberg, USA

3943. Fisher, W. A. P., Fatigue aspects of structural design, *J. roy. aero. Soc.* **60**, 543, 198–202, Mar. 1956.

Author points out advantage of designing aircraft wings as statically indeterminate structures so that more than one element must fracture before catastrophic failure can occur. Such design permits detection of incipient fatigue failure in a part, and its replacement, before other parts have suffered damage. Representative *S-N* curves for specified probabilities of failure are given and the use of such data in design for a specified life is explained.

By some, author might be considered over-sanguine as to attainable accuracy; in this connection, the conclusions of Kepert and Payne, *NACA TM* 1397, are of interest.

R. J. Roark, USA

3944. Dubuc, J., and Welter, G., Investigation of static and fatigue resistance of model pressure vessels, *Welding J.* **35**, 7, 329s–337s, July 1956.

3945. Kingery, W. D., and Pappis, J., Note on failure of ceramic materials at elevated temperatures under impact loading, *J. Amer. ceram. Soc.* **39**, 2, 64–66, Feb. 1956.

3946. Matthaes, K., Stress corrosion (in German), *Z. Metallk.* **47**, 1, 37–42, Jan. 1956.

3947. Topping, A. D., Rock strength: The conditions of failure, *World Oil* **141**, 4, 109–110, 112, 114, 117, Sept. 1955.

Rock is ordinarily considered a brittle material, but many experiments have shown that rock does yield plastically under certain conditions of triaxial loading. Though we may be interested in rupture, we must consider yielding first, especially since a material may yield without failure even though its rupture strength is no higher than its yield strength.

The two most successful theories for the yielding of ductile metals have been the maximum shear theory and the distortion energy or octahedral shear theory. Neither the maximum shear nor the octahedral shear theory are satisfactory for rock. Both can be modified to give better agreement with test results by making the critical value of the shear a function of the normal stress on its plane. The Mohr theory of strength, which is perhaps the most widely used for brittle materials, may be regarded as such a generalization of the maximum shear theory. Attempts to verify the theory experimentally for the strength of sandstone were, however, unsuccessful. A similar generalization of the distortion energy theory was proposed by Schleicher [*ZAMM* **6**, p. 199, 1926; also **5**, p. 478, 1925] and Nadai [*J. appl. Mech.* **1**, 3, 111–129, 1933], wherein the octahedral shearing stress $\tau_{y.}$ is a function of the octahedral normal stress σ. Unlike Mohr's theory, this takes into account the influence of the intermediate principal stress. An analysis of the same tests which refute the Mohr theory, however, has shown that they agree little, if any, better with the Schleicher-Nadai theory [Topping, A. D., "Elastic, elastic-plastic, and viscoelastic states of stress and strain around a vertical cylindrical hole in a semi-infinite gravitational body," Ph.D. Theses, Univ. Illinois, 1951].

If, as an approximation, strength is assumed to vary linearly with hydrostatic pressure, the Mohr theory can be expressed as

$$\sigma_3 + q\,\sigma_1 = c \qquad [1]$$

where $\sigma_1 > \sigma_2 > \sigma_3$ (algebraically). Analogously, the generalized theory is with the same approximation

$$\sigma_3 + b\,\sigma_2 + q\,\sigma_1 = c \qquad [2]$$

In these equations, σ_1, σ_2, and σ_3 are principal stresses and b, q, and c are experimental constants. The question is, can Eq. [1] or [2] also be used as a fracture criterion?

Fracture may be either a sliding failure or a cleavage failure. A brittle material tested in simple tension will fracture by cleavage in compression by sliding along an inclined plane. One may be led to associate sliding fracture with a limiting octahedral shearing stress or strain, and cleavage fracture with a hydrostatic tension or volumetric strain. This suggests a fracture criterion in terms of strains.

Equation [2] can be modified for fracture as follows

$$\epsilon_3 + u\epsilon_2 + j\,\epsilon_1 = k \qquad [3]$$

where ϵ_1, ϵ_2, and ϵ_3 are the principal strains, and u, j, and k are experimental constants.

In applying yield and fracture criteria to conditions in a bore hole, there is another factor to be considered. When triaxial tests are made in which the lateral pressure is exerted by fluid having free access to the surface of the specimen, the axial load to cause failure appears to be independent of the lateral pressure. If the specimen is jacketed (with rubber, say) so that the lateral pressure is still effective but the fluid cannot penetrate the specimen, the

lateral pressure has a very important influence. This has been explained by Terzaghi [*Proc. ASTM* **45**, p. 777, 1945] from considerations of the granular structure of rock, which cannot be gone into here. E. Siebel, Germany

3948. Baker, J. F., and Tipper, C. F., The value of the notch tensile test, Inst. mech. Engrs., Prepr. 13 pp.

Paper is arranged in four main parts with respect to contents. Part I deals with development of a notch tensile test procedure as it is used in the Engineering Department, Cambridge University, (England). One of the basic features of this test is that normally the test strip shall have the full plate thickness so that eventual brittleness of the plate in its natural dimensions can be revealed. Part II describes investigations of steel plates, from storage tank and ships, that have shown brittleness and failure under service conditions. These investigations bring evidence that a notch tensile test gives fairly good information on whether a normal fibrous or a brittle fracture can be expected to initiate at the service temperature. Comparison with Charpy V-notch- and Izod-test results from the same casualty material gives an indication that a criterion of brittle-safeness based upon level amounts of energy absorption might be dangerous. Part III deals with experiments in which notch tensile test is used to predict service performance of specimens of larger size and more complicated form. Authors announce more investigation work upon the effect of geometry, but they are principally convinced that notch tensile test is of great value.

Part IV deals with considerations upon the criticism that has been made on the interpretation of the test, and this part forms a highly interesting statement as to the level of investigation to which present-day work for solving the brittle-fracture problem is raised. R. Nilson, Sweden

3949. Watkins, J. M., Spangler, R. D., and McKannan, E. C., Stress relaxation in concentrated solutions of polymethyl methacrylate: molecular weight dependence, *J. appl. Phys.* **27,** 7, 685–690, July 1956.

The measurement of stress relaxation in concentrated solutions of polymethyl methacrylate has revealed a relation between the spectrum of relaxation times, the weight avergae molecular weight, and the molecular weight distribution. The spectrum of relaxation times and the essentially identical theories developed by Rouse, Bueche, and Zimm are used to calculate an average relaxation time which is directly proportional to the weight average molecular weight. It is then shown that a first approximation to the distribution of molecular weights may be calculated if one assumes that the friction factor increases with the 1.5 power of the molecular weight. This distribution for selected samples of polymethyl methacrylate is compared with the Schulz distribution in which the parameters were obtained from light scattering and osmotic pressure experiments. The ratios M_z/M_w calculated from the theory and stress relaxation data on a rough fraction, a whole polymer, and a blend of two whole polymers are found to be 1.3, 1.7, and 1.9, respectively. From authors' summary by H. Mii, Japan

Material Test Techniques

(See also Revs. 3948, 4139)

Mechanical Properties of Specific Materials

(See also Revs. 3878, 3945, 3947, 3948, 3949, 3863, 4068, 4105, 4122, 4123)

3950. Forscher, F., A theory of the yield point and the transition temperature of mild steel, *J. appl. Mech.* **23,** 2, 219–224, June 1956. See AMR **9,** Rev. 1821.

3951. Baron, H. G., Stress/strain curves of some metals and alloys at low temperatures and high rates of strain, *J. Iron Steel Inst. Lond.* **182,** part 4, 354–365, Apr. 1956.

Comparative tensile tests have been carried out on a number of metals and alloys at temperatures of 20, −78, and −196 C, using strain rates of 10^{-3} and 10^{+2} in./in./s. Stress/strain curves are presented.

The strain rate and temperature have a marked effect on iron and the softer ferritic steels. In dynamic tests below a critical temperature, all the ferritic materials exhibit a type of premature failure in which the strain is entirely localized to a short neck. It is shown that this is a consequence of the adiabatic nature of this test. The variation of upper yield stress with temperature and strain rate appears to obey an activation-energy relationship over part of the range of stress. Within this range, the activation energy for yielding varies with the stress in the manner predicted by Cottrell and Bilby. The divergence from theory at low stresses may be due to simultaneous strain-aging, while, at high stresses, brittle fracture or twinning eventually intervene before the upper yield point is reached.

An aluminum alloy of the Duralumin type showed a yield point which varied with strain rate and temperature in a similar manner to the steels, but anomalies which are again probably caused by a strain-aging process were found in slow tensile tests at 20C. With annealed tough-pitch copper, the proof stress appears to be little affected by strain rate and temperature but the rate of strain-hardening is strongly affected by both. The strain rate has relatively little influence on the behavior of annealed brass.

From author's summary

3952. Nish, J. I., Choice, properties and behaviour of high temperature materials, *J. Instn. Engrs. Austral.* **28,** 4/5, 109–116, Apr./May 1956.

At high temperatures the behavior pattern of a material can be notably different from its behavior at room temperature. Some of the properties and characteristics of materials that have a bearing on their usefulness at high temperature are discussed. Directions in which better high-temperature materials are being sought are indicated. From author's summary

3953. Baron, J., Today's tendencies in the evaluation of mechanical strength of refractory products at high temperatures (in Italian), *Metallurgia ital.* **48,** 1, 10–14, Jan. 1956.

The mechanical strength at high temperatures of silica products can be evaluated through the under-load sinking test, considering the point of rupture, provided that the heating rate of the test piece is consistent with its size.

The results of a double set of tests, the one carried out according to French specifications, the other with the French standard test piece but with a heating rate reduced to 1°/ min at temperatures higher than 1550C, are identical to ± 10C for the rupture temperature, which confirms the value of the test.

The sinking test being insufficient, however, some under-load creep tests at constant temperature have been carried out. The results related with two Cowper recuperator collars confirm Dale's law, when the homogeneity of the product investigated is sufficient. Under the most severe testing conditions, creep seems to depend mostly upon the porosity, while under most favorable conditions it seems to depend mainly upon the refractoriness.

A set of tests carried out on a low-porosity collar, featuring a high creep resistance, confirm perfectly Dale's law; that is, the logarithm of deformation is a linear function of temperature.

From author's summary

3954. Fielding, J., Rubber pressing, *J. Inst. Metals* **84,** 147–159, Feb. 1956.

The reasons why rubber pressing is used for the forming of aircraft components are given, and the methods and equipment for form-

ing aluminum and magnesium alloys and titanium are described. Deformations possible with straight and curved flanges, and the effects of heat treatment, flange design, and sheet thickness are outlined, with the object of providing information for the designer. The merits of "cold" and "hot" forming of magnesium alloys and titanium are discussed, and details of hot-pressing techniques are given with special reference to die materials, temperatures, pressures, and heat-resisting rubbers.

It is concluded that rubber pressing is a very satisfactory, economical, and well-proved process for forming many aircraft parts in aluminum alloys. With magnesium alloys, it has been discovered that, by hot forming at 300C, greater deformations are possible than with aluminum alloys, and very accurate components, exactly to die shape, can be produced. For titanium, too, hot pressing at 300C is preferred, excellent results being obtained, particularly on sheets 0.036 in. thick and less. With thicker sheets the much higher pressures required create difficulties.

From author's summary

3955. Leadabrand, J. A., and Norling, L. T., Simplified methods of testing soil-cement mixtures, *Highway Res. Board Bull.* **122,** 35–47, Jan. 1955.

Plasticity, Forming and Cutting

(See also Revs. 3938, 3939, 4113, 4114, 4120)

3956. Ross, E. W., Jr., On the ideally plastic indentation of inset rectangular bands, *J. appl. Mech.* **23,** 2, 244–246, June 1956.

The indentation of an inset rectangular band by a smooth flat die is considered. An upper bound for the pressure required to produce indentation is determined. Its value depends upon the dimensions of the die and band and, as may be anticipated, is less than the upper bound for plane strain, $(2+\pi)k$.

The derivation is based on Tresca's yield condition and two limit-design theorems. W. Schroeder, USA

3957. Conway, H. D., The indentation of a transversely isotropic half-space by a rigid punch, *ZAMP* **7,** 1, 80–85, 1956.

Analogies in structural foundation problems exist in the indentation concept of a transversely isotropic mass indented by a cylindrical punch. Paper is meant to show that if the distribution of the contact pressure is known under any axisymmetrical punch acting on an isotropic half-space, the corresponding distribution for a transversely isotropic half-space may be written down by inspection. Effects of frictional forces under the punch are neglected and the plane is assumed to be deformed solely by normally applied forces.

Paper giving analysis of problem is preceded by a brief historical introduction on the work of Boussinesq, Love, Schubert, Harding and Sneddon, and others. S. K. Ghaswala, India

3958. Shaw, M. C., Smith, P. A., Cook, N. H., and Loewen, E. G., Leaded steel and the real area of contact in metal cutting, ASME Semiann. Meet., Cleveland, O., June 1956. Pap. 56-SA-37, 14 pp. + 7 figs.

Paper deals with the influence of lead addition on the machining characteristics of a low-carbon, cold-drawn, resulfurized open-hearth steel from a more fundamental point of view than found in literature heretofore published on the same subject. Analysis shows that leaded steel is characterized by the *smallness* of the amount of lead present (0.25% by weight), the size of the individual particles (approximately 0.0001 in.), mean spacing of the surface particles (about 0.002 in.), and the average thickness of lead layer spreading out at the area of real contact (some 25 atom layers of lead).

Cutting tests were performed at low speeds of 0.5 and 10 fpm on steels with and without lead, and calculations reveal that the real area of contact at the tool-chip interface is only 1 to 2% of the apparent area of contact and that about ¼ of the real area of contact is coated with lead. This latter finding, together with the extremely low shear strength of lead, is believed to be instrumental in understanding the free-machining characteristics of leaded steels. Test results on tool face frictional force and specific cutting energy for both leaded and unleaded steel using eight different cutting fluids are presented and discussed. However, the detailed mechanism of certain improvement in cutting-fluid performance in the presence of lead has not been clarified.

B. T. Chao, USA

3959. Shaw, M. C., Smith, P. A., Cook, N. H., and Loewen, E. G., The influence of lead on metal-cutting forces and temperatures, ASME Semiann. Meet., Cleveland, O., June 1956. Pap. 56-SA-36, 15 pp.

The effects of additions of lead and sulphur to steel upon tool-face friction F, cutting energy per unit volume u, and tool-face temperature θ were investigated. By means of high-speed flash pictures of chips in the process of being formed, it was found that lead was effective in decreasing chip thickness at all speeds, but particularly at high speeds. Sulphur, on the other hand, was found most effective in this regard at tool temperatures (speeds) below about 900 F. It was found that certain cutting fluids (principally cutting oils) interfered with the action of lead and sulphur, and fluids that are normally effective on ordinary steels may give poorer results than air upon steels containing lead or sulphur. In all cases, leaded steels were found to give lower values of F, u, and θ, than comparable nonleaded steels and the improvement due to lead was far greater than could be achieved from cutting fluids at speeds of 150 fpm and above.

The paper ends with a note of warning that the results on F (friction force), u (energy per volume of metal), and θ (tool face temperature) cannot be used to predict tool life directly, and an example is presented which illustrates this point.

From authors' summary by A. O. Schmidt, USA

3960. Takeyama, H., Murai, T., and Usui, E., Speed effect on metal machining (I), *J. mech. Lab. Tokyo* **1,** 2, 59–61, 1955.

In the theoretical formulas of metal machining which have been worked out by many authorities, generally speaking the speed effect is not taken into consideration. This is one of the reasons why the formulas do not agree well with practical cases.

In this report chip formation and rupture in the shear zone at various speeds were investigated photographically, and great differences in mechanism of metal cutting with variations of cutting speed were pointed out. Furthermore, the chips, turned out at various speeds, were carefully examined, and chip-thickness ratios and shear angles were figured out. By using these data, some empirical formulas in relation to chip-thickness ratio, shear angle, rake angle, and cutting speed were established. These agree with practical cases very well. From authors' summary

3961. Sokolovskii, V. V., Some remarks concerning a plane problem in the theory of plasticity (in Russian), *Prikl. Mat. Mekh.* **18,** 6, 762–763, 1954.

In a problem of sheet-drawing [R. Hill, "Mathematical theory of plasticity", 1946 and 1950], a situation may occur when a line of velocity discontinuity along the slip lines terminates at a point which is different from the exit corner A of the die, and which is an inadmissible point of discontinuity. This situation takes place when the tangent stress t is smaller than the plastic constant k, and when the point of the velocity discontinuity B at the wall of the sheet is located on segments between such slip lines on which velocity discontinuity terminate in A. These segments decrease away from A, and vanish for a most common case when $t = k$.

R. M. Evan-Iwanowski, USA

3962. Fortier, A. L. A., Sonic pneumatic measurements of lengths and their applications, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap 55-A-111, 8 pp. + 5 figs.

Air gages are nowadays extensively used in production. These gages, owing to the remarkable properties of sonic flows, permit measurement of lengths with a high degree of accuracy and reliability. The air gages, making use of sonic flows and of the principle of the pneumatic Wheatstone bridge, are, in addition, particularly well suited for automatic machine tool controls.

From author's summary

Hydraulics; Cavitation; Transport

(See also Revs. 3970, 4126, 4129, 4155, 4160)

Book—**3963. Manual of hydraulic engineering [Spravochnik po gidrotekhnike]**, Moscow, Gosizdat, 1955, 828 pp. $5.75.

This large handbook, compiled by leading Russian experts, contains fundamental information for design of hydraulic structures, is based on official standards and practical experience, and is intended for use of designing engineers. Book does not include hydroelectric plants, irrigation and drainage, fishing problems, sea ports, water supply and sewerage, arc dams, tunnels, river regulation. All information is brief, but many tables and nomograms are given for direct use. Costs are not discussed at all.

General content of the manual: Mathematical tables and few formulas; Hydraulic design: hydrostatics, flow through orifices, weirs, uniform and nonuniform flow in open channels, hydraulic jump, spillways, culverts; Strength of materials; Elements of structures: girders, frames, slabs, flexible base; Structure settling on clay foundations; Materials applied in hydraulic engineering: steel and cast-iron, lumber, stone, cement and lime, mortar and concrete, seals; General standards for design; Timber constructions; Steel constructions; Concrete and reinforced-concrete constructions; Investigations; Hydrometric work; Hydrology; Geological survey; Soil properties; Hydraulic constructions; Reservoirs; Basic data for dam design; Seepage computation; Concrete gravity dams, without and with spillway; Reinforced-concrete and buttressed-concrete dams; Timber dams; Earth dams; Rock-fill dams; Bulkheads; Spillways and outlets; Gates and valves; Canals and their lining; Aqueducts, syphons, culverts; Penstocks; Inlets; Silt settlers; Ship locks; Log and raft gates; Honeycomb structures with metal sheet piling.

As compared with our handbooks, content of this manual is more limited; however, it has many detailed tables, e.g., for slabs, frames, steel profiles. A table of noneroding velocities for various soils and different water depths is of particular interest, as are many runoff maps also. Manual clearly represents contemporary developments in the technique of hydraulic engineering in Russia. Bibliography is given after every chapter, exclusively in Russian language. All foreign names are carefully eliminated, although some methods are mentioned anonymously. It is strange to an American reader that cumbersome title of prize-winner is repeated in full on every page of discussion, e.g., p. 532–534.

S. Kolupaila, USA

Book—**3964. Scimemi, E., Compendium of hydraulics [Compendio di Idraulica]**, Padova, Casa Editrice Dott. Antonio Milani, 1955, 328 pp.

This comprehensive work has now been published as a book, after several editions had already appeared in simple unbound form. The author, who died in 1952, had originally planned it as a reference book for the students of the Padova University (Faculty of Engineering). The good and complete descriptions of flow problems, numerical examples, and, last but not least, the many numerical tables and graphs have met with a favorable reception.

Besides the theoretical principles the book contains a great many of those empirical facts which are indispensable in hydraulics. For instance, coefficients for inlet and outlet losses derived from existing plants and model experiments are quoted; in another passage the mathematical results of Allievi and Camichel are compared with those of a water-hammer experiment. For computing the movement of water level in surge tanks, a numerical example (diminution of load) is given of a system with two surge tanks, one behind the other. In computing backwaters there is particular reference to the methods of Bresse and Bakhmeteff, and to the approximative formula of Marzolo. The final chapters describe wave movements, underground flow, model conditions, and methods of measurements.

The clearness of the descriptions will foster the book's popularity.

P. Franke, Germany

3965. Escande, L., and Nougaro, J., Unsteady flow in a head race canal leading to a pressure tunnel (in French), *Houille blanche* **11**, 2, 240–249, May/June 1956.

Two particular problems of unsteady motion, with occurrence of waves in an open channel related to a pressure tunnel, are reported. Authors apply the familiar momentum equation and solve it by means of step integration. Reviewer believes that this procedure is advantageous in this case. First, authors solve the common problem of a canal leading to a pressure gallery which is followed by a surge tank. Authors check their analytical results with model tests and find good agreement for the surge-tank water elevations and for the surge in the canal.

As a second application, authors solve the problem of a siphon located between two canals. In this case, authors combine the analytical with the graphical method as a modification of Bergeron's procedure. Here also, good agreement was found with model studies.

A. Balloffet, USA

3966. Zababurin, I. A., Hydraulic loss in a circular sluice gate (in Russian), *Gidrotekh. Stroit.* 25, 2, 41–43, 1956.

An empirical formula is derived for a local loss

$$c_f = a + 3.70\,(1/s - 1)^2$$

where $s = b/(d + 2r)$, b is the opening height, d is the diameter of culvert, r is the radius of entrance roundings, $a = 0.75$ for a high sill, or $a = 0.35$ for a sill lower than diameter d.

S. Kolupaila, USA

3967. Levi, I. I., Local scour in the tailwater (in Russian), *Gidrotekh. Stroit.* **25**, 1, 48–50, 1956.

A further contribution to the much discussed problem [AMR **8**, Rev. 1171; **9**, Revs. 479, 480, 2243]. Formula by Vyzgo is theoretically incorrect; if used as an empirical one it must be improved by experimental coefficients. Author prefers a formula by D. I. Kumin and also his own.

S. Kolupaila, USA

3968. Bibikov, D. N., Sludge ice distribution over river depth (in Russian), *Gidrotekh. Stroit.* **25**, 3, 39–40, 1956.

Logarithmic law of sludge distribution along a vertical is applied and integrated. A diagram shows the sludge concentration to the desired depth of a river for flow computation of floating ice over a spillway.

S. Kolupaila, USA

3969. Arsenishvili, K. I., Surface wave criteria in sloping channels (in Russian), *Gidrotekh. Stroit.* **25**, 3, 41–44, 1956.

After reviewing several formulas, author offers two conditions when surface wave will not occur: (1) $0.02 > s > 0.30$, where s is the channel slope; and (2) $h_0/P_0 \geq 0.10$, where h_0 is water depth, P_0 is wetted perimeter, both at uniform flow.

S. Kolupaila, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 3963, 3965, 4000, 4004, 4007, 4011, 4027, 4033, 4072, 4073, 4077, 4127, 4132, 4133, 4155, 4158, 4161)

Book—**3970. Milne-Thomson, L. M., Theoretical hydrodynamics, 3rd ed.**, New York, The Macmillan Co., 1956, xxi + 632 pp. $9.80.

Engineers and physicists working in the field of fluid mechanics hardly require an introduction to this distinguished book. An excellent outline of its contents may be found in the preface.

The work is unique in that vector methods and notation are consistently employed. Consequently, author believes that the reader "–will be amply rewarded by a physical insight into the phenomena under discussion which are, in general, made unnecessarily obscure by expression in particular coordinate systems." This is probably a matter of taste, in view of the fact that lucid and enjoyable books in this field have been written using Cartesian coordinates.

The book consists of the following 20 chapters: Bernouilli's equation; Vectors; Equations of motion, two-dimensional motion; Complex variable; Streaming motions; Aerofoils; Sources and sinks; Moving cylinders; Theorem of Schwartz and Christoffel; Jets and currents; Helmholtz motions; Rectilinear vortices; Waves; Stokes' stream function; Spheres and ellipsoids; Solid moving through a liquid; Vortex motion; Viscosity; Subsonic and supersonic flow. It may be seen that the coverage is quite extensive. As a result, some of the topics are of necessity treated rather briefly (for example, chap. 20, on subsonic and supersonic flow).

The third edition is essentially the same as the second with the exception of new material on the topics of virtual mass, reflection across free streamlines, and potential flow with a free surface.

"Theoretical Hydrodynamics" is recommended highly, especially to the research worker. I. I. Glass, Canada

3971. Beran, M., A note on laminar axially symmetric jets, *Quart. appl. Math.* **14,** 2, 213–214, July 1956.

It is customary to solve the boundary-layer and Navier-Stokes equations for jets by assuming a solution of the form $\psi = rf(\theta)$. It is shown that solutions of the Navier-Stokes equation of this form cannot vanish on the surface of a cone and it is claimed this is a serious drawback.

Reviewer cannot agree with this claim. A formal solution is obtained containing three arbitrary constants. For a certain choice of values, the solution describes a jet. It would be convenient if, for other values of the constants, useful solutions were obtained; but their absence does not invalidate the jet solution.

W. Squire, USA

3972. Curtet, R., A study of eddies created by a plane jet issuing into a bounded uniform stream (in French), *C. R. Acad. Sci. Paris* **241,** 24, 1705–1708, Dec. 1955.

A theory developed by M. Craya and the author [title source, **241,** p. 621, 1955] is employed to determine the nature of the reverse eddy resulting when a plane jet issues into a uniform stream which is bounded by parallel walls. Good agreement between experiment and theory is obtained.

Failure to define symbols required reviewer to read all prior references. J. P. Craven, USA

3973. Kalikhman, L. E., Transformation of the equations of the turbulent boundary layer in a gas to linear ordinary differential equations and their solution in finite form (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **106,** 3, 401–404, 1956 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass.).

The integral momentum relation and integral energy relation are obtained by integrating across the boundary-layer equations of motion and energy. These equations are transformed into two linear ordinary differential equations by introduction of proper variables. Their solutions are given in finite form, and problems of boundary layer can be solved without special assumption on the velocity and temperature fields or the mechanism of turbulence.

M. Kataoka, Japan

3974. Fraenkel, L. E., On the flow of rotating fluid past bodies in a pipe, *Proc. roy. Soc. Lond.* (A) **233,** 1195, 506–526, Jan. 1956.

The mathematical expression is sought for the flow of a rotating, incompressible fluid through contractions and expansions of a pipe, or past bodies of revolution on pipe axis. Flow for upstream is assumed to have constant axial velocity and constant angular velocity about pipe axis. Author finds a wave motion downstream. He has used following mathematical resources with great skill: A generalized Stokes' stream function, complex integration, and developments into series of Bessel functions. Interesting results are shown in tables and drawings. Leaving out viscosity, author has oversimplified the problem. Therefore, the suppositions are not the only possible ones, but reviewer believes that they are reasonable.

H. O. Faxén, Sweden

3975. Schlichting, H., Laminar flow about a rotating body of revolution in an axial airstream, *NACA TM* 1415, 26 pp. + 20 figs., Feb. 1956.

See AMR **7,** Rev. 1560.

3976. Csanady, G. T., The rise of a hot smoke plume, *Austral. J. appl. Sci.* **7,** 1, 23–28, Mar. 1956.

The approximate solution of Sutton (1953) for the path of hot smoke is simplified and expressed in nondimensional form. The resulting equation is compared with observations recorded by Bosanquet, Carey, and Halton. It is concluded that a nondimensional parameter, called here "Sutton number", plays a role similar to that of the Reynolds number in pipe flow; also, that at larger distances from the source, Sutton's approximate solution is verified to a remarkable extent, whereas closer to the source, a slightly modified version of the formula may be used.

From author's summary

3977. Welanetz, L. F., A suction device using air under pressure, *J. appl. Mech.* **23,** 2, 269–272, June 1956.

See AMR **9,** Rev. 1146.

3978. Kemp, N. H., and Sears, W. R., On the wake energy of moving cascades, *J. appl. Mech.* **23,** 2, 262–268, June 1956.

See AMR **9,** Rev. 1923.

3979. Verkamp, J. P., and Williams, S. L., Testing nuclear-plant leaktightness, *Nucleonics* **14,** 6, 54–57, June 1956.

Compressible Flow, Gas Dynamics

(See also Revs. 3970, 4002, 4004, 4005, 4011, 4018, 4031, 4032, 4034, 4037, 4039, 4071, 4075)

3980. Eichelbrenner, E. A., Methods of computing the two-dimensional laminar boundary layer in the compressible region (in French), *ONERA Publ.* 83, 135 pp., 1956.

Author summarizes methods for computing incompressible and compressible two-dimensional laminar boundary layers, noting that most methods are approximate and yet do not permit evaluation of errors involved. He then proposes two methods for treating the compressible case. The first replaces the equations of motion by finite difference equations. This method permits evaluation of the error involved. The second method utilizes a hodograph transformation (a la L. Crocco) and represents an almost rigorous integration of the boundary-layer equations. The latter method has no restrictions on the free-stream and wall boundary conditions.

These methods are compared with an integral method due to E. Gruschwitz. Critical points of the boundary layer are discussed. The boundary-layer development in the presence of attached and detached shocks is also considered. H. Mirels, USA

3981. Busemann, A., On the gas dynamics of a rotating impeller, *NACA TM* 1385, 16 pp., Mar. 1956. (Translation from ZAMM **18,** 1, Feb. 1938.)

It is shown that, for a compressible flow with constant entropy,

the pressure rise maintains the direct relation to the circulation around the blades existing for incompressible flow. In contrast, however, the torque, and with it the power consumption, is increased because of sound waves already traveling to infinity at subsonic circumferential speeds.

From author's summary

3982. Keller, J. B., Spherical, cylindrical and one-dimensional gas flows, *Quart. appl. Math.* **14,** 2, 171–184, July 1956.

The problem of spherical, cylindrical, or planar unsteady flows of a polytropic gas is formulated in Lagrangian variables. The class of solutions considered describes motions of the particles as product of a function of time and of another function of the Lagrangian variable. Assuming at time zero a special pressure distribution, one can interpret one of these solutions as the expansion of an isentropic or nonisentropic gas sphere into vacuum. Other solutions correspond to the propagation of shocks.

G. Guderley, USA

3983. Bevierre, P., and Diringer, P., Theoretical study of resistance of a tapered and sharpened body to advance movement started instantaneously in fluid at rest (in French), *Rech. aéro.* no. 49, 5–10, Jan./Feb. 1956.

Paper represents an extension of the methods developed by A. von Baranoff dealing with the determination of the resistance of a body of revolution under uniform acceleration or deceleration. The present work is concerned with the case of a body set in motion instantaneously at a specified velocity. The potential function representing an appropriate distribution of sources and sinks is developed in terms of three limiting quantities. These limits are calculated as dependent on the velocity range, that is, subsonic, sonic, or supersonic.

The remainder of the paper is concerned with a calculation of the resistance by integration of the pressure which is, in turn, obtained from Bernoulli's equation for compressible unsteady flow. A numerical example completes the work.

M. J. Thompson, USA

3984. Palmer, P. J., The lift on wings at sonic speeds by means of an electrical resistance analogue, *J. roy. aero. Soc.* **60,** 542, 137–139 (Technical Notes), Feb. 1956.

This note shows how a pure resistance analog can be used to find the lift on low-aspect-ratio wings traveling, with small incidence, at speeds close to the sonic velocity.

The method is applicable to flat, twisted, or cambered wings and is simple in operation; the results obtained being in close agreement with those obtained by calculations based on the same theory.

From author's summary

3985. Heaslet, M. A., and Spreiter, J. R., Three-dimensional transonic flow theory applied to slender wings and bodies, *NACA TN* 3717, 72 pp., July 1956.

This highly mathematical treatment of the nonlinear, approximate differential equation for subsonic and supersonic potential flow derives the integral equations of transonic flow from Green's theorem with the inclusion of discontinuity surfaces to represent shock waves. Theory is applied to slender wings and bodies; in particular, a slender elliptic cone-cylinder. Transonic similarity rules for lift and drag coefficients are confirmed. Some experimental data on cone-cylinders and wings is introduced which checks theory at $M=1$.

R. E. Street, USA

3986. Sakurai, T., The flow past a flat plane accompanied with an unsymmetric dead air at Mach number 1, *J. phys. Soc. Japan* **11,** 6, 710–715, June 1956.

Flow over flat plate at an angle of attack at Mach one is computed by means of the hodograph under the assumption that a dead air region with sonic pressure arises downstream of the plate.

G. Guderley, USA

3987. Watts, G. A., An experimental investigation of a sonic jet directed upstream against a uniform supersonic flow, *Univ. Toronto Inst. Aerophys. TN* 7, 19 pp. + 16 figs. + 19 plates + 2 tables, Jan. 1956.

Flow configurations generated by directing sonic jets of air, helium, and sulphur hexafluoride upstream into a uniform supersonic ($M=2.5$) flow of air were studied experimentally (schlieren and interferograms).

The jets expanded to a high Mach number, passed through a curved shock wave, and then decelerated subsonically, approaching the interface between the jet and the main flows. The main flow formed a detached bow wave. The resulting flow configurations were steady.

From author's summary by J. Lukasiewicz, Canada

3988. Keune, F., and Oswatitsch, K., The law of equivalence and similarity laws for transonic flow and drag of bodies of zero angle of attack and low aspect ratio (in German), *ZAMP* **7,** 1, 40–63, 1956.

Ward has shown that the formula for the pressure drag coefficient of the pointed wing of low aspect ratio in supersonic flow is the same as that for the pointed body of revolution when expressed in terms of the cross-sectional area at any point. Aim of this paper is to obtain the generalized law of equivalence of Ward's law for transonic flow.

This paper shows in its first part that the velocity disturbance on the pointed wing of law aspect ratio moving at transonic velocity can be reduced to the velocity disturbance on the pointed body of revolution with the same distribution of cross-sectional area. And from the similarity law for the pointed body of revolution, the corresponding similarity law for the pointed wing of low aspect ratio is derived. Further, the law of equivalence for the pressure drag is given, which is a generalization for transonic flow of Ward's law for supersonic flow.

T. Okamoto, Japan

3989. Schafer, M., On the continous return of disturbed supersonic flows to the subsonic regime in mixed flow fields (in German), *J. rational Mech. Analysis* **5,** 2, 217–250, 1956.

Paper points out that there exist subsonic flows with imbedded supersonic region wherein the transition from supersonic to subsonic flow is continuous. Investigation attempts to answer whether such flows represent an isolated phenomenon or whether there exist neighboring solutions for slightly modified boundary conditions in which the transition from supersonic to subsonic flow remains continuous.

Starting from a known mixed flow and altering the boundary slightly in the supersonic flow region, neighboring solutions are obtained. It is not, however, known whether these are analytic functions in the complete flow field.

The interplay of wall effects prohibits such a continuous transition in channels.

S. Ostrach, USA

3990. Cabannes, H., Tables for the determination of detached shock waves (in French), *Rech. aéro.* no. 49, 11–15, Jan.-Feb. 1956.

Subject problem is the study of the detached shock obtained when a uniform supersonic stream flows axially over a body of revolution with a rounded nose. The double power series expansion procedure developed by the author in previous papers [see e.g., AMR **5,** Rev. 781] is now carried out to a sixth-order approximation for free-stream Mach number equal to 2. Tables and graphs are presented from which the shock can be obtained in terms of the body parameters and vice versa.

Paper raised some doubts for reviewer. In addition to obvious

misprints, some random checking located other less obvious ones. A randomly chosen tabular entry did not agree with the corresponding result computed from the equations presented in the text. Throughout the text the same symbol is used for the radius of curvature of the nose of the body and the radius of curvature of the apex of the shock. In addition, convergence questions would seem to be serious [see Lin and Shen, *NACA TN* 2506; AMR **5**, Rev. 1467]. The magnitude of the higher-order terms would indicate very slow convergence at best. No comparison has been made with previously obtained fourth-order approximation results and experimental results. P. Chiarulli, USA

3991. Love, E. S., Pressure rise associated with shock-induced boundary-layer separation, *NACA TN* 3601, 32 pp., Dec. 1955.

An examination is made of recent contributions in the field. For laminar boundary-layer flow, von Doenhoff's incompressible formula as modified by Stewartson's transformation is used by author for velocity ratio at separation and compared with corresponding results of Loftin and Wilson, Stewartson and Howarth. For turbulent boundary layers, author chooses as basis of comparison the methods of Reshotko and Tucker, and of Mager. In view of practical applications, author connects peak pressure rise with separation instead of pressure at separation. Experiments of the author confirm above theoretical methods.

Reviewer thinks comparison of various methods in Fig. 7 of paper is not free from objections: The use of Stewartson's transformation for predicting compressible *turbulent* boundary-layer separation from incompressible values is doubtful. Further, the accurate epxerimental results of Gadd, Holder and Regan [*Proc. roy. Soc. Lond.* (*A*) **226**, 227–253, 1954; AMR **8**, Rev. 1782] are not considered. H. Schuh, Sweden

3992. Pocock, P. J., The calculation of the wave drag of an arbitrary slender body by means of an electrical analogy tank, *Nat. aero. Establ. Canad. LR*–127, 23 pp. + 12 figs., Mar. 1955.

In view of the analogy between the supersonic wave drag of a slender body and the subsonic induced drag of an "equivalent" lifting line (first drawn to attention by von Karman), the electric analogy tank developed by Malavard for solving the Prandtl lifting-line equations can be used to calculate the wave drag of arbitrary slender bodies.

A Malavard-type electric analogy tank at the National Aeronautical Establishment is described and its use illustrated by some example problems. From author's summary

3993. Licher, R. M., Optimum two-dimensional multiplanes in supersonic flow, *Douglas Aircr. Co. Rep.* SM-18688, 23 pp., Jan. 1955.

Biplanes and other multiplanes are studied by linear theory to find optimum configurations having minimum drag due to lift in supersonic flight. It is shown that, by use of suitable camber, the drag of a two-dimensional biplane can often be reduced (as much as 25% in one case) compared to that of the same biplane without camber.

Since in nonplanar systems there is usually interference between lift and thickness, it is sometimes possible to further reduce the drag below that of the cambered biplanes by introduction of a thickness distribution. A general criterion is developed to indicate when the optimum distribution of camber and thickness for minimum drag due to lift has been obtained. A number of examples of optimum configurations having both camber and thickness are shown.

Distributions of camber and thickness having no net lift or drag, called zero loadings, are discussed; one familiar example of such a zero loading is the Busemann biplane at zero angle of attack. It is shown how zero loadings may be combined with optimum distributions of camber and thickness to provide more depth for structure without increasing the drag. From author's summary

3994. Nicolas, J., Flow around protuding points in supersonic air intakes (in French), *Rech. aéro.* no. 49, 23–26, 1956.

Experimental tests have led to an empirical profile for protruding points in supersonic air intakes which yield very high efficiencies. A method is presented for the calculation of the flow about such points with a view to guiding subsequent experimental research on the shape of supersonic air intakes.

From author's summary by C. E. Carver, Jr., USA

3995. Fenain, M., and Valée, D., Application of the theory of homogeneous flows to the calculation of lift effects in sweptback wings (in French), *Rech. aéro.* no. 50, 17–25, 1956.

The theory of homogeneous supersonic flow fields developed by Germain [title source, no. 7, 3–16, Jan. 1949; AMR **3**, Rev. 103] is applied here to problems of lifting wings having supersonic trailing edges. The results of Germain's work are reviewed briefly. Formulas for lift, drag-due-to-lift, rolling moment, and pitching moment are worked out for wings with subsonic leading edges. For wings with supersonic leading edges, the lifting problem is identical with the drag problems worked out in an earlier paper by the present authors [title source no. 44, 9–41, Mar. 1955; AMR **9**, Rev. 1519]. Application is made to delta wings and sweptback delta (swallow-tail) wings having incidence and parabolic twist (but not camber) within the homogeneous-flow restriction. Results are presented in a series of graphs for delta wings and for one family of sweptback delta wings; relative leading-edge sweep is the abscissa.

Finally, the total drag is evaluated for these two families, the two constants of the parabolic twist distribution being selected for optimum ratio of $drag/(lift)^2$. It is shown that this ratio can always be reduced relative to that of a flat wing by use of this type of twist. The advantage is greater for wings with subsonic than supersonic leading edges. W. R. Sears, USA

3996. Conrad, O., Flow in Laval nozzles in the region of low supersonic velocities (in German), *Maschinenbau-Technik* **4**, 8, 395–403, Aug. 1955.

A method of designing the throat section of axially symmetrical and of rectangular supersonic nozzle is discussed.

In the case of an axially symmetrical nozzle, first the Mach line is drawn according to equation developed. Upstream from the Mach line the nozzle profile follows a parabola (equation given) which reaches well into the region of subsonic flow. Downstream the profile deviates from the parabola, meeting the diverging part of the nozzle which is constructed by the method of characteristics.

The throat section of a rectangular supersonic nozzle is designed directly by the method of characteristics after substituting the characteristics diagram with a numerical table.

The error involved in this method is shown to be negligible.

K. Pilarczyk, USA

3997. Roper, G. M., Calculation of the effect of camber and twist on the pressure distribution and drag on some curved plates at supersonic speeds, *Aero. Res. Counc. Lond. Rep. Mem.* 2794, 68 pp., 1955.

The present investigation was undertaken with a view to finding the shape of a curved wing, such that the thrust loading on the leading edges, particularly near the wing tips, is removed or modified. The shapes of two groups of such wings have been found:

(1) For the first group, when the wings are at design incidence, there are no leading-edge pressure singularities, and therefore no leading-edge thrust. The pressure difference is finite and positive everywhere on the wing, and decreases to zero on the leading edges.

(2) For wings of the second group, the leading-edge singularity is modified so that its strength increases along the edge from zero at the apex to a maximum, and then decreases to zero, after which it would become negative. The effect of additional incidence is to increase the local lift everywhere and to move the positions of

aximum and zero singularity strength further downstream.

In this report, it is also shown how the shapes of wings of the second group can be determined to satisfy certain requirements with respect to camber and twist, or the magnitude of aerodynamic characteristics.

The lift, the induced drag, and the pitching-moment coefficients for some wings of triangular planform have been calculated, and the results are shown graphically.

From author's summary by A. J. Eggers, Jr., USA

3998. **Fraenkel, L. E., Supersonic flow past slender bodies of elliptic cross-section,** *Aero. Res. Counc. Lond. Rep. Mem.* 2954, 7 pp., 1955.

The theories of supersonic flow past slender, smooth, pointed bodies of arbitrary cross-sectional shape, due to Ward, and of the flow past slender bodies of revolution with discontinuities in tangential slope, due to Lighthill, are applied and extended to calculate first approximations for the aerodynamic forces on bodies of elliptic cross section with discontinuities in profile slope. Open-based bodies are included in this class, but only the external forces are considered. The investigation is restricted to bodies the major axes of whose cross sections are coplanar and whose cross sections have constant eccentricity. General expressions are deduced for the wave drag, lift, induced drag, and pitching moments of such bodies.

The drag formula bears a marked resemblance to that for the equivalent body of revolution (i.e., the body of revolution with the same axial distribution of cross-sectional area), but the discontinuities introduce a slight difference in one term. The lift formula is identical with that already deduced by Ward for a particular case of the present problem. The general theory is applied to elliptic cones, and a comparison is made with Squire's solution of this problem. Numerical results for the wave drag of bodies of revolution having straight and parabolic profiles are also extended to bodies of elliptic cross section.

From author's summary by M. H. Bertram, USA

3999. **Ferri, A., and Nucci, L. M., Theoretical-experimental analysis of low-drag supersonic inlets having a circular cross section and a central body at Mach numbers of 3.30, 2.75, and 2.45,** *NACA Rep.* 1189, 37 pp., 1954.

In the design of supersonic inlets for ramjets and turbojets, it is desirable to obtain a maximum pressure recovery with a minimum aerodynamical drag. At high Mach numbers, high pressure recovery is generally obtained only at the expense of high external drag. Authors have investigated nose inlets with partially external and partially internal supersonic compression. These inlets were designed to have low external drag. A conical shock is employed for external compression. Tests were made of a number of combinations of cone angle, central body diameter, cowling shape, and central body position relative to the cowling. Pressure recoveries of 0.57, 0.67, and 0.78 were obtained at Mach numbers of 3.30, 2.75, and 2.45, respectively. Results may be used to evaluate the minimum possible external drag for a given external diameter of the body containing the ramjet or turbojet and a given size of free-stream tube for the entering flow. The proportion between internal and external diffusion for minimum drag of a given inlet having a specified ratio of free-stream tube to maximum diameters may be determined.

E. L. Foster, USA

Wave Motion in Fluids

(See also Revs. 3965, 3969, 4158)

Turbulence, Boundary Layer, etc.

(See also Revs. 3972,3973,4046,4053,4076,4104,4132,4136)

Book—4000. Townsend, A. A., The structure of turbulent shear flow, New York, Cambridge University Press, 1956, xii + 315 pp. $7.50.

Book is the first comprehensive treatment of practical flow problems based on the modern statistical theory of turbulence. Treatment is very clear and a nice balance of theory and experiment. The relative simplicity of the mathematical apparatus employed is surprising. The collection of experimental data (an appreciable fraction of which are work of author and his collaborators at Cambridge) is sufficient justification for the book.

First chapter introduces the concept of correlation and spectrum functions. The scope of the others is indicated by the titles: (2) The equations of motion in shear flow; (3) Flows without appreciable variation of mean velocity; (4) Uniform distortion of homogeneous turbulence; (5) Free turbulence: Unrestricted shear flow; (6) Free turbulence: Motion of the large eddies; (7) The turbulent wake; (8) Turbulent jets; (9) Turbulent flow in pipes and channels; (10) Boundary-layer flow without pressure gradient; (11) Boundary-layer flow in a pressure gradient; (12) Shear flow between rotating cylinders.

Key chapter is the fifth, in which the basic concepts of Reynolds number similarity, self-preservation, and turbulent transport are introduced. These are used to analyze specific flow problems in chaps. 7–12. The stress is on a basic understanding rather than detailed calculations. A logical foundation is provided for such well-known results as the constant eddy viscosity approximation for wakes and jets and the logarithmic velocity distribution near walls.

As author states in the introduction, no attempt is made to discuss alternate treatments. Reviewer believes that at least a critical bibliographical survey of the field should have been included. However, as we can expect an encyclopedic treatment in Volumes IV and V of the Princeton Series, this is not too serious an omission.

Reviewer believes this book will be indispensable to workers in the field and interesting to everyone who has to deal with a fluid mechanical problem. It will undoubtedly have a profound influence on future developments in the field.

W. Squire, USA

4001. Chandrasekhar, S., Theory of turbulence, *Phys. Rev.* (2) **102**, 4, 941–952, May 1956.

It is pointed out that if there are aspects of the turbulence phenomenon which are truly universal, then they should be capable of being characterized in terms of the parameters ϵ and ν which denote the constant rate of dissipation of energy per unit mass and the kinemetic viscosity, respectively, and these two parameters only, without reference either to the mean square velocity $(u_1^2)_{Av}$ or to the size of the largest energy containing eddies. This is a slight modification of Kolmogoroff's similarity principles as currently formulated. It appears that

$$\chi = \partial\psi(r,t)/\partial r, \qquad \text{where } \psi(r,t) = \tfrac{1}{2}[(u_1' - u_1'')^2]_{Av}$$

and u_1' and u_1'' are the velocities in the x-direction (say) at two points on the x-axis separated by a distance r and at times an interval t apart, can be so specified. The similarity principles require that if this is the case, χ should be of the form $\chi = (\epsilon^3/\nu)^{1/4} X(r(\epsilon/\nu^3)^{1/4}, t(\epsilon/\nu)^{1/2})$, where X is a universal function of the arguments specified. In the limit of zero viscosity, χ must have the more special form $\chi \longrightarrow r^{-1/3}\sigma(t/r^{2/3})$ $(\nu \longrightarrow 0)$.

The boundary conditions on $\sigma(x)$ are that $\sigma = \sigma_0$ (> 0) and $d\sigma/dx = 0$ as $x = 0$, and $\sigma \longrightarrow 0$ as $x \longrightarrow \infty$.

It is shown that with a slight modification of the premises of the theory described previously, an equation for χ can be derived which is compatible with the requirements of the similarity principles as formulated. In particular, the ordinary differential equation for σ to which the theory leads can be solved. The solution for σ which is found satisfies all the boundary conditions of the problem and is unique, apart from adjustable scale factors. The

predicted evolutions of χ and the vorticity correlations are illustrated.

From author's summary by L. M. Grossman, USA

4002. Krzywoblocki, M. Z., Spatially homogeneous turbulence of a compressible fluid (in French), *Publ. sci. tech. Min. Air, France* no. 314, 54 pp., 1956.

Highly mathematical. Author extends to compressible fluids the statistical methods developed by Kampé de Fériet for the statistical treatment of homogeneous isotropic turbulence of an incompressible fluid which clarified greatly the kinematics of this turbulence without, however, being able to do much with the dynamics of it. In the case of compressible fluids one must consider not only the spectral distribution of the kinetic energy, but of three functions involving (1) the absolute magnitude of the velocity, (2) the density, and (3) the temperature. Assuming the Navier-Stokes equations to be satisfied, author derives the fundamental equation for the spectrum of the kinetic energy.

Reviewer believes this to be an important contribution to the statistical theory of turbulence.

D. ter Haar, England

4003. Kraichnan, R. H., Pressure field within homogeneous anisotropic turbulence, *J. acoust. Soc. Amer.* **28**, 1, 64-72, Jan. 1956.

Paper has a bearing on sound generated within an aircraft by pressure fluctuations of the turbulent boundary layer on its walls. Integral expressions are derived for root-mean-values, correlation and moments of correlation of the pressure. In case of isotropic turbulence, correlation is always positive and falls to zero faster than the velocity correlation. With anisotropic turbulence, pressure correlation is also negative in places and possesses a long range "tail." With the idealized models of turbulence treated, anisotropy was found to reduce mean-square-pressure fluctuations for a fixed mean kinetic energy of turbulence, but it is as yet uncertain whether this result may be generalized.

H. Schuh, Sweden

Book—4004. Goertler, H., and Tollmien, W., 50 years of boundary layer research—anniversary publication composed of original contributions [50 Jahre Grenzschichtforschung—Ein Festschrift in Originalbeiträgen], Braunschweig, Friedr. Vieweg & Sohn, 1955, viii + 499 pp. DM 56.

It is fifty years since Prandtl published his theory of flow in the neighborhood of solid boundaries, and to mark the occasion Professors Goertler and Tollmien have put together this collection of new papers. Invitations to contribute a paper on boundary-layer theory, this term being interpreted very broadly, were distributed widely in a number of countries, and the published volume represents the response, together with a brief general account of the development of boundary-layer theory by Professor Tollmien. Half the total number of 44 papers are from authors in U.S.A., one quarter from authors in Germany, and the remainder from various other countries. On the whole, the papers are a striking demonstration of the extent of progress with boundary-layer theory, and of the many and varied applications of the underlying idea of a balance between viscous and inertia forces confined to certain narrow layers.

The quality of the work is rather uneven, as might be expected with so heterogeneous a collection of authors. Quite apart from the specialized scientific content of the papers—of which there is much that is valuable, although difficult to summarize, even in very general terms—the book has a good deal of interest for the student of the current scientific scene. It is not a systematic account of what we now know under the heading of boundary-layer theory, but it can presumably be regarded as a fair indication of the nature of work in progress at the expanding frontier of the subject. It is clear that Prandtl's idea has been abundantly fruitful, that the last word on the fundamental analysis has not yet been said, and that large numbers of people find the task of applying the theory congenial.

G. K. Batchelor, England

4005. Bader, R., and Gilles, J., Problem of semi-infinite plate in viscous flow (in French), *Rech. aéro.* no. 49, 17-22, 1956.

Within the framework of linearizing hypotheses proposed by Lagerstrom, Cole, and Trilling, authors derive the exact expression for the boundary-layer displacement thickness on a semi-infinite flat plate. In fact, this became the problem of solving by the Wiener-Hopf method two integral equations set up by the authors. This rigorous calculation agrees remarkably well with the result obtained from the iteration proposed by Lagerstrom, Cole, and Trilling.

From authors' summary by S. Corrsin, USA

4006. Kalikhman, L. E., Turbulent boundary layer in gas flow past a plate (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **106**, 1, 23-26, 1956 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 6 pp.).

Assuming the temperature field, density field, frictional strain, and specific flow of heat properly, author has obtained the stagnation temperature and velocity profile. On the other hand, author finds another expression of velocity profile utilizing the Newton and Fourier formulas. From these two equations the drag formula is obtained. Comparisons of the calculated coefficient of resistance with experimental data without heat exchange and with heat exchange show comparatively good agreement. Velocity profile differs from experiments in some degree.

Reviewer believes that the improvement of velocity profile will bring a better result for larger Mach number.

M. Kataoka, Japan

4007. Cheng, S.-I., and Elliot, D., The unsteady laminar boundary layer on a flat plate, 1956 Heat Transfer and Fluid Mechanics Institute, Stanford, Cal., Prepr. 14, 18 pp.

Authors discuss the boundary layer on a flat plate occupying the positive half of the x axis when the mainstream velocity u_e is a function of the time t only and $u_e(0) = 0$. A series expansion centered on the leading edge is developed and they claim that their solution possesses the correct behavior at small t or large x.

The principal interest in this problem is mathematical, the engineering aspect being of less importance, and from this standpoint the reviewer believes that the paper is open to a serious objection. The solution must satisfy specified boundary conditions at $x = 0$, $t \geqslant 0$ and at $t = 0$, $x \geqslant 0$. The series solution developed by the authors takes no account of the boundary condition at $t = 0$, and so we cannot expect it to be correct when t is sufficiently small. Any agreement with the correct and known solution here must be regarded at present as fortuitous. If, instead, an expansion is centered on $t = 0$ it is known that the solution independent of x is correct when $x \geqslant \int_0^t u_e\, dt$. Sin-I Cheng [unpublished: see ref. 2 of the present paper] attempted to extend the solution to smaller values of x by means of a series, but failed. The principal reason is again that the solution in this region depends on the boundary condition at $x = 0$ as well as on the velocity profile at

$$x = \int_0^t u_e\, dt.$$

K. Stewartson, England

4008. Townsend, A. A., The diffusion behind a line source in homogeneous turbulence, *Proc. roy. Soc. Lond.* (A) **224**, 1159, 487-512, July 1954.

The central theme of this investigation is an accelerated molecular conduction associated with turbulent diffusion of heat from a line source in a field of homogeneous turbulence. The experimental setup consists of a fine heated wire, acting as a line source, in a field of turbulence produced by an upstream grid. Measurements are made of mean temperatures in the wake, temperature fluctuations, products of temperature fluctuations at two

points, and products of temperature and cross-stream velocity fluctuations. Theoretical relationships are derived, based on Taylor's theory of turbulent diffusion by continuous movements, and results are analyzed so as to show the results in a most informative manner and also to bring to light the accelerated molecular conduction caused by turbulent distortion of the heated sheet and consequent increase in local temperature gradients. Under certain conditions this effect invalidates the assumption usually made that molecular conduction and turbulent transport occur independently. In particular, accelerated molecular conduction becomes so large in the later stages of the diffusion that it is hardly possible to calculate the single particle Lagrangian correlation.

A number of other interesting features of turbulent diffusion are brought out, including similarity and self-preserving processes during decay. G. B. Schubauer, USA

4009. Walz, A., New approximative method for compressible boundary layer (in French), *C. R. Acad. Sci. Paris* **243**, 1, 15–16, July 1956.

4010. Sato, H., Experimental investigation on the transition of laminar separated layer, *J. phys. Soc. Japan* **11**, 6, 702–709, June 1956.

Transition of a two-dimensional separated layer was investigated in detail. Mean velocity distributions of the laminar layer showed a similar profile and agreed with Lin's calculation. In the transition region, hot-wire anemometer survey of velocity fluctuation revealed the existence of sinusoidal waves, the frequency of which lay in the unstable zone predicted by stability theory. Transition points determined by layer thickness and by fluctuation measurements were in good coincidence. A simple empirical relation was derived for transition distance.

From author's summary by J. V. Becker, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 3933, 3983, 3985, 3988, 3993, 3995, 3997, 4035, 4038, 4043, 4047, 4051, 4055, 4074, 4091, 4104, 4136, 4156)

4011. Farren, W. S., The aerodynamic art, (The 44th Wilbur Wright memorial lecture), *J. roy. aero. Soc.* **60**, 547, 431–449, July 1956.

4012. Kuhn, R. E., Investigation at zero forward speed of a leading-edge slat as a longitudinal control device for vertically rising airplanes that utilize the redirected-slipstream principle, *NACA TN* 3692, 33 pp., May 1956.

Results are presented of an investigation of a leading-edge slat as a longitudinal control device for vertically rising airplanes in hovering flight. The model consisted of a semispan wing equipped with a 60-percent-chord and a 30-percent-chord slotted flap operating in the slipstreams of two large-diameter propellers. The tests were conducted at zero speed in a large room. The results indicate that the slat can provide increments of pitching moment of the order of those required for trim and control in hovering flight. Also the leading-edge slat can appreciably reduce the adverse effects of the ground on the effectiveness of the slotted-flap wing in redirecting the slipstreams downward.

From author's summary

4013. Kuhn, R. E., and Spreemann, K. P., Preliminary investigation of the effectiveness of a sliding flap in deflecting a propeller slipstream downward for vertical take-off, *NACA TN* 3693, 25 pp., May 1956.

Results are presented of an investigation of the effectiveness of a sliding flap in deflecting propeller slipstreams downward for vertical take-off. The model consisted of a semispan wing equipped with a 50-percent-chord sliding flap and a 25-percent-chord plain flap. The tests were conducted at zero forward speed in a large room and included the effects of flap deflection, proximity to the ground, a leading-edge slat, and a chord extension. The results indicate that the diving moments associated with the sliding flap were appreciably smaller than those of the slotted-flap configuration previously investigated.

From authors' summary

4014. Horne, W. B., Smiley, R. F., and Stephenson, B. H., Low-speed yawed-rolling characteristics and other elastic properties of a pair of 26-inch-diameter, 12-ply-rating, type VII aircraft tires, *NACA TN* 3604, 98 pp., May 1956.

The low-speed (up to 2 mph) cornering characteristics of two 26 × 6.6, type VII, 12-ply-rating tires under straight-yawed rolling were determined over a range of inflation pressures and yaw angles for two vertical loads. The cornering characteristics of one tire rolling along circular paths of different radii were investigated for one condition of vertical load and inflation pressure. Static tests were also performed to determine the vertical, lateral, torsional, and fore-and-aft elastic characteristics of the tires. The quantities measured included lateral or cornering force, drag force, torsional moment or self-aligning torque, pneumatic caster, vertical tire deflection, lateral tire deflection, wheel torsion or yaw angle, rolling radius, and relaxation length. Some supplementary tests were made which included measurements of tire footprint area and the variation of unloaded tire radius with inflation pressure. From authors' summary

4015. Boucher, R. W., and Kraft, C. C., Jr., Analysis of a vane-controlled gust-alleviation system, *NACA TN* 3597, 45 pp., Apr. 1956.

Analog computer study on step-gust response of airplane with automatic, vane-controlled, interconnected flap-elevator alleviation system. Several system configurations studied center around an optimal condition based on static equilibrium in steady flow. Due to noninstantaneous uniform flow immersion of the whole system, this optimal condition is not realized, viz., rotational effect of flap deflection at vane initiation; flap down wash impulse on tail; wing downwash-gust impulse on tail. Appreciable rotational accelerations and velocities result. There is some alleviation of translatory gust effect at expense of equivalent translatory flap effect. All studies are for uniform gust gradient and path speed and stationary-flow force coefficients. Indications are that adequate stability and controllability are realizable without adversely affecting attainable "optimal" alleviation. Promising prospects if system could follow gust and downwash impulses more closely. Misprint appears in main flap moment coefficient value.

P. R. Hardesty, USA

4016. Kraft, C. C., Jr., Initial results of a flight investigation of a gust-alleviation system, *NACA TN* 3612, 19 pp., Apr. 1956.

A flight investigation of a gust-alleviation system designed to improve passenger comfort in flight through turbulent air has been recently initiated. The system is designed to maintain constant lift in rough air by means of trailing-edge wing flaps operated by an automatic control system which is controlled by signals from an angle-of-attack vane located at the nose of the airplane. The gust-alleviation system is at least capable of reducing the normal accelerations due to gusts by 50% in the range of the natural frequency of the airplane. From author's summary

4017. Heyson, H. H., and Katzoff, S., Normal component of induced velocity in the vicinity of a lifting rotor with a nonuniform disk loading, *NACA TN* 3690, 45 pp., Apr. 1956.

Part I presents a method for computing, from available calculations for uniform disk loading, the effect of nonuniform circularly symmetrical disk loading on the normal component of induced velocity in the vicinity of a lifting rotor. Charts of the normal component of induced velocity are given for the longitudinal plane of symmetry and for the major axes of rotors with two different, nonuniform, circularly symmetrical disk-load distributions. Part II develops certain symmetry relations for the induced velocities in

the plane of a uniformly loaded rotor and also develops relations between the radial load distribution of the rotor and the radial variation of induced velocities in the wake.

From authors' summary

4018. Ferri, A., and Clarke, J. H., On the use of interfering flow fields for the reduction of drag at supersonic speeds, *Polyt. Inst. Brooklyn, Aero. Lab. Rep.* 304, 50 pp. + 22 figs., Mar. 1956.

Drag reduction at supersonic speeds by judicious use of wave interference is discussed in general. The interference between a three-dimensional swept wing and a small reflecting surface is considered in detail. Using linearized theory, the interference drag can be evaluated without determining the flow field. Several biplane configurations are treated and it is shown that the wing drag can be greatly reduced or the volume near the wing root can be increased without penalty in drag. Three-dimensional biplane is discussed from practical point of view.

G. V. R. Rao, USA

4019. Harrin, E. N., Comparison of landing-impact velocities of first and second wheel to contact from statistical measurements of transport airplane landings, *NACA TN* 3610, 22 pp., Feb. 1956.

A statistical analysis of vertical velocities at second wheel to touch was made from photographs of 353 landings of transport airplanes at Washington National Airport. The effects of gust, rolling direction, and number of engines are presented. Comparisons are made with vertical velocities of the first wheel to touch [*NACA TN* 3194].

The results of the statistical analysis indicated that the probability of equaling or exceeding a given high value of vertical velocity was slightly greater in any given landing for the second wheel than for the first wheel to contact. Gusty wind conditions had the effect of increasing the vertical velocities for both the first and second wheels to contact. The effect of the direction of rolling was such that the probability of equaling or exceeding a given high value of vertical velocity was greater for the wheel toward which the airplane was rolling just prior to initial impact. There appeared to be no correlation between the relative vertical velocities of first and second wheels to contact and the ratio of wheel tread to radius of gyration, such as would be expected from theoretical considerations. From author's summary

4020. Povolny, J. H., Steffen, F. W., and McArdle, J. G., Summary of scale-model thrust-reverser investigation, *NACA TN* 3664, 49 pp., Feb. 1956.

An investigation was undertaken to determine the performance and other characteristics of several basic types of thrust reversers. Models of three types, target, tail-pipe cascade, and ring cascade, were tested with cold flow. The effects of design variables on performance and reversed-flow boundaries, along with thrust-modulation characteristics, were determined.

Values of reverse-thrust ratio as high as 84% could be obtained if the gas were turned far enough and permitted to attach to the cowling. Even without reversed-flow attachment and with controlled reversed-flow patterns, all three types of reversers gave reverse-thrust ratios over 40%. The ring type with a 7° cowl gave 41%, the target type with a 7° cowl gave 49%, and the tail-pipe-cascade type with a 0° cowl gave 73%. All three types had satisfactory thrust-modulation characteristics. Thus it was concluded that the selection of a reverser type may be based primarily on installation considerations rather than on performance characteristics. From authors' summary

4021. Dreher, R. C., A method for obtaining statistical data on airplane vertical velocity at ground contact from measurements of center-of-gravity acceleration, *NACA TN* 3541, 21 pp., Feb. 1956.

A method is presented whereby the contact vertical velocities of airplanes are obtained on a statistical basis from measurements of maximum incremental center-of-gravity acceleration. In order to evaluate this method, 268 landings with a small trainer airplane have been made. The vertical velocities obtained are compared with the actual vertical velocities obtained from a trailing-arm velocity indicator. The comparison is in the form of probability curves, and the difference in the two curves is less than 0.2 fps throughout the range of velocity covered in this investigation. This close agreement of the two sets of data indicates that a reliable probability curve for vertical velocity was obtained from the maximum incremental center-of-gravity accelerations. A limited amount of data for a medium bomber airplane is presented; these data indicate that the method is probably applicable to large airplanes. From author's summary

4022. Atkins, P. B., Flight testing of a Vampire aircraft at maximum lift, *Aero. Res. Labs. Melbourne, Austral. Note* ARL/F23, 17 pp. + 26 figs., Aug. 1955.

Flight tests have been made on an Australian Vampire in order to assess the effect on boundary-layer separation of the fitting of turbulence strips.

These tests extend into flight a wind-tunnel investigation of the mode of stall of this airfoil and consist of (a) an investigation of the general flow over the wing tip by means of tufts; (b) a pilot's assessment of the change in aircraft behavior at stall due to the fitting of the turbulence strips; (c) detailed measurements of wing surface pressures near the leading edge; (d) comparative measurements of aircraft drag; and (e) a boundary-layer traverse just ahead of the aileron with and without the strips fitted.

The turbulence strips were found to make the aircraft stall behavior consistent at all altitudes, and this is reflected in changes in the pressure distribution near the wing leading edge. Thickening of the boundary layer at stall near the leading edge of the control surface is reduced by the addition of the strips.

No measurable increase in aircraft total drag was caused by fitting turbulence strips.

There are indications that the high-speed lift characteristics are greatly improved with the turbulence strips fitted.

From author's summary

4023. Cicala, P., Optimum performance of an airplane (in Italian), *Monogr. Lab. Aero. Politecn. Torino* **89,** 361, 9 pp., 1954–55.

A general analysis of the optimum performance of aircraft and missiles is presented from a variational standpoint. Following hypotheses are used: The aircraft is a particle of variable mass; the flight path is contained in a vertical plane; the acceleration of gravity is a constant; the aerodynamic lag is disregarded; the thrust is a function of velocity, altitude, and engine mass flow.

The variational problem is formulated within the general frame of the problems of Mayer type, i.e., as the problem of minimizing the difference ΔG between the values assumed by a function G of the variables of the problem at the end points.

By the use of Lagrange multipliers, the system of Euler equations is derived. The boundary conditions are formulated, in general, by means of the so-called transversality condition. The Legendre-Clebsch conditions, the Weierstrass condition, and the Jacobi condition are analyzed and discussed. The Erdmann-Weierstrass corner conditions are introduced in connection with problems involving discontinuities. Simplified solutions are presented for the particular case where the centripetal accelerations are neglected.

It is the opinion of the writer that Professor Cicala's paper is an important and valuable contribution to the general theory of the optimization of the performance of aircraft and missiles.

A. Miele, USA

4024. Payne, P. R., Induced aerodynamics of helicopters, *Aircr. Engng.* **28,** 324, 46–51, 54, Feb. 1956; **28,** 325, 82–87, Mar. 1956; **28,** 326, 123–129, Apr. 1956; **28,** 327, 148–153, May 1956.

In principle, author reviews the state of art of basic rotor aerodynamics. It starts with application of simple momentum theory to vertical and forward flight, then induced velocity fields in forward flight are considered theoretically and compared with test data. Vertical drag is discussed in hovering, ascending, and forward flights. This is followed by considerations of ground effect and then author goes back to vertical flight regime by introducing strip theory to analysis of this regime of flight. Some practical applications of the latter are demonstrated by showing ways of obtaining optimum twist and planform of blades. So-called exact performance equations are developed and practical application of them through performance computer is suggested. Aerodynamic rotor interference of tandems in hovering and in forward flight is considered and theoretical interpretation of this phenomenon is outlined. Concluding article of this series is devoted to slipstream rotation of unshrouded rotor, then power thrust aspects of ducted fans are considered, and, finally, discussion reverts back to vortex theory of unshrouded rotors.

W. Z. Stepniewski, USA

4025. **Katzenberger, E. F., and Rich, M. J., An investigation of helicopter descent and landing characteristics following power failure,** *J. aero. Sci.* **23,** 4, 345-356, Apr. 1956.

Paper is primarily concerned with an analytical investigation of helicopter descent and landing characteristics following partial or complete power failure on take-off.

An analysis is made of the time history of descent velocity following power failure for the critical case of vertical take-off. The investigation includes the determination of the time rate of change of descent velocity and velocity at ground contact, employing pilot control inputs representative of pilot capabilities and helicopter characteristics representative of current helicopters.

The special case of steady-state autorotative descent and landing is treated separately, and an index for comparing helicopter designs as to autorotative landing capabilities is derived.

From authors' summary

4026. **Black, J., Flow studies of the leading edge stall on a swept back wing at high incidence,** *J. roy. aero. Soc.* **60,** 541, 51-60, Jan. 1956.

The flow separation on a sweptback wing with 44 degrees leading-edge sweep at 18 degrees incidence has been investigated by means of detailed pressure-distribution measurements over the leading edge, boundary-layer flow determination with liquid film technique, and yawmeter traverses. A wool-tuft grid was also used, and a spin detector was developed to search for regions of vorticity. These tests established that the flow separates on the leading edge; over the inboard sections it re-attaches behind a "short" separation bubble, while outboard it only re-attaches near the trailing edge, thus forming a "long" separation bubble, or else fails to attach. The separated flow in what has been commonly called the tip stall does, in fact, take the form of a "ram's horn" vortex with the origin, or "tip," located at the junction of the two bubbles on the leading edge. The vortex lies outward across the wing surface at approximately 20 to 25 degrees to the line-of-flight before curving aft to be shed into the wake, where it extends almost from mid semispan to the wing tip. This vortex induces considerable changes in flow direction, both on and over the wing, and also in the wake. Thus in the wake, a maximum downwash of 23 degrees is induced aft of the mid semispan, and there is an upwash of 17 degrees at the outer edge of the vortex, almost aft of the tip. A good correlation between yawmeter results and the boundary-layer flow direction indications from the liquid-film technique was obtained. From author's summary

4027. **Guennoc, H., Calculation of symmetric airfoil in incompressible flow** (in French), *Rech. aéro.* no. 50, 13-15, 1956.

A rapid procedure, based on a paper by R. Legendre [title source, no. 22, 3-6, 1951; AMR **5,** Rev. 795], is presented for calculating the shape and velocity distribution of a class of airfoil sections for which certain properties such as the leading-edge radius, trailing-edge angle, position where the velocity is maximum, and thickness-chord ratio can be specified.

G. E. Nitzberg, USA

4028. **Puffert, H. J., Interference between wing, fuselage, and tail unit in oblique flow** (in German), *Z. Flugwiss.* **3,** 9, 323-331, Sept. 1955.

Interference between airplane parts is derived by considering effects of transverse flow on fuselage upon flow over wing and tail assembly, and consequent modifications of wing-vortex system which, in turn, affect flow over fuselage and tail assembly and the forces upon them. Approximate formulas for the forces and moments due to these interference effects are derived and compared with available wind-tunnel data. The agreement is surprisingly good in spite of the fact that boundary-layer effects upon the flow over the tail surfaces had not been considered.

L. Landweber, USA

4029. **McCormick, B. W., The effect of a finite hub on the optimum propeller,** *J. aero. Sci.* **22,** 9, 645-650, Sept. 1955.

Goldstein's classical theory of lightly loaded propellers, which was applied to the circulation distribution on six- and eight-bladed propellers by the reviewer and Janakiram [AMR **7,** Rev. 255], has been extended by the author to include the effect of the hub. Calculations are made to show that, for a given pitch, the introduction of a hub increases the value of the bound circulation. This increase decreases with increase in the number of blades. The effect of hub sizes on the circulation distribution has been calculated for two-bladed propellers (circulation for zero hub radius given by Goldstein) and for eight-bladed propellers (circulation for zero hub radius given by Acharya and Janakiram). In addition, the ratio of circulation with and without a hub has been calculated and plotted against the number of blades with r/R and $\omega r/v$ as parameters.

Reviewer feels that the paper is an interesting contribution to the literature on lightly loaded propellers.

Y. V. G. Acharya, India

4030. **Barbieux, J., Contribution to the study of the effect of wall constraint in two-dimensional incompressible flow** (in French), *Publ. sci. tech. Min. Air, France* no. 304, 247 pp., 1955.

Author reexamines the problem of wall interference on two-dimensional wind-tunnel tests of airfoils. First portion of the work is a critical review of existing theories and experiments on wall-interference effects. Here author discusses discrepancies among various existing theories, shortcomings of various experimental investigations, and lack of conclusive agreement between theory and applicable experiments.

In second portion of paper, author first develops a new theory, next describes an elegant and carefully performed set of measurements on an airfoil in a wind tunnel of variable test section height, and, finally, compares the experimental results with the new theory. This new theory is developed along classical lines; the airfoil is represented by its replacement potential, and floor and ceiling of the wind tunnel are represented by an infinite series of suitably placed image potentials. Correction formulas are deduced by appeal to thin airfoil theory, and are valid for airfoil chord to wind-tunnel-height ratios up to a value of unity in incompressible flow, and up to a value of $1\text{-}M^2$ in subsonic compressible flow. In contrast to earlier theories, corrections are found in terms of the unconfined theoretical flow over a comparison airfoil which has the same thickness and the same circulation per unit chord per unit velocity as the airfoil in constrained flow. Thus, the formulas involve theoretical values of lift coefficient, moment coefficient, and lift curve slope, which all must be assumed or calculated. On the other hand, earlier methods of calculation, as,

for example, theory of Allen and Vincenti [*NACA TR* 782], express corrections in terms of quantities measured in the wind tunnel. However, author's method has theadvantage that a knowledge of the profile drag is not required, whereas it is essential that drag be measured in order to apply the method of Allen and Vincenti.

Pressure distributions and profile drags were measured by author on a NACA 63015 airfoil having a chord of 0.30 m. The airfoil spanned a specially constructed wind-tunnel test section provided with an adjustable floor and ceiling. The test section was 6.40 m long, 0.31 m wide, and height could be varied from 0.30m to 0.97m. Maximum velocity was 35 mps. When corrected by author's method, the integrated pressure data from these tests produced unique section-lift curves, section-moment curves, section-drag polars, and pressure distributions for all chord-to-height ratios in the range that was explored.

Appendix I of paper gives interesting results of a series of incidental blockage experiments carried out by author. He shows that measured drag coefficients of various sizes of circular and rectangular plates mounted normal to the stream can, when referred to the corresponding drag coefficients in free air, be correlated as a function of the ratio of plate area to test section area, the correlation being independent of Reynolds number.

Prospective users of the results of this paper should be warned that several typographical errors exist in the mathematical development. Unfortunately, these cannot be cited conveniently because the equations are unnumbered. H. A. Stine, USA

4031. Multhopp, H., Methods for calculating the lift distribution of wings (Subsonic lifting-surface theory), *Aero. Res. Counc. Lond. Rep. Mem.* 2884, 96 pp., 1955.

Paper represents a valuable contribution to the theory of airfoils covering the case of a lifting surface of arbitrary planform. The extension from the lifting line to the lifting surface concept is discussed in some detail, following the previous work of the author on the earlier problem.

The key to the success of the method from a computational standpoint lies in the careful selection of pivotal points for satisfying the downwash equations and determining the chordwise distributions. Charts are provided which facilitate this operation.

The methods are adapted to wings of any planform, including variations in taper, sweep back, and aspect ratio. The effect of section irregularities, such as flaps and ailerons, may also be included. Several examples are worked out and the results compared with other methods. M. J. Thompson, USA

4032. Tucker, W. A., A method for the design of sweptback wings warped to produce specified flight characteristics at supersonic speeds, *NACA Rep.* 1226, 17 pp., 1955.

One of the problems connected with the sweptback wing is the difficulty of controlling the location of the center of pressure and, hence, the pitching moment. A method is presented for designing a wing to be self-trimming at a given set of flight conditions. Concurrently, the spanwise distribution of load on the wing is made to be approximately elliptical, in an effort to maintain low wing drag.

These flight characteristics are achieved by warping the wing out of a plane. The required warp is determined by the values of the coefficients of a four-term series describing the pressure distribution; these values in turn are determined from four conditions on the lift, pitching moment, and spanwise load distribution.

The method is directly applicable to several wing planforms, including the triangle and the sweptback planform with finite tips, under the restriction that the leading edge must be subsonic and the trailing edge must be supersonic. The application to any specific problem is simplified to a routine computational procedure by the presentation of certain basic data in tabular form. A discussion is given of some points to be considered in the application of the method to a practical case, and several representative examples are worked out. The resulting wings are shown to be ones which might practicably be built.

From author's summary by M. Finston, USA

4033. Truitt, R. W., and Martin, F. W., Pressure distribution about two-dimensional diamond airfoils and a 180° wedge, *Virginia Polyt. Inst. TN OSR-TN*-54-149, 25 pp. + 11 figs., June 1954.

Paper dervies, by the Schwarz-Christoffel mapping theorem, the pressure distribution about two-dimensional diamond airfoils for the zero angle-of-attack condition. The theoretical (incompressible) pressures are compared with corresponding low-speed wind-tunnel tests and the agreement is excellent except at the shoulder. The zero pressure coefficient was found to move toward the shoulder with increasing wedge angle. The effect of afterbody configuration is discussed in view of the theoretical and experimental results. The theoretical pressure distribution over a 180° wedge in a bounded and unbounded stream is presented. The 180° wedge is found to be less sensitive to wall influence than the small-angle wedges. Experimental low-speed results for the 180° wedge in an unbounded stream are found to be such that free streamlines spring from the shoulder. The sound-space theory is applied to the experimental results for the 180° wedge, and the local Mach number distribution is found to be "symmetric" about free-stream Mach number one, but not constant.

From authors' summary

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 3882,4049,4054)

4034. Berman, J. H., Lift and moment coefficients for an oscillating rectangular wing-aileron configuration in supersonic flow, *NACA TN* 3644, 46 pp., July 1956.

The problem of a partial-span rectangular control surface, hinged at its leading edge and harmonically oscillating in supersonic flow, is treated by linearized flow theory. The velocity potentials for a wing-aileron configuration performing vertical translation, wing pitching, and aileron rotation are derived as power series in terms of a frequency parameter to the fifth power of the frequency of oscillation. Expressions for force and moment coefficients at any spanwise location are presented. These expressions, derived for an undisturbed wing, are applied to a flutter study at $M = 1.3$ of a rectangular wing-aileron configuration with the wing distorted in bending and torsion. Different spanwise positions of the aileron are investigated. An important conclusion is that aileron mass balance may have an adverse effect on flutter at low supersonic Mach numbers. A. I. van de Vooren, Holland

4035. Goland, L., and Perlmutter, A. A., A comparison of the calculated and observed flutter characteristics of a helicopter rotor blade having both control system and blade flexibility, Princeton Univ., Dept. Aero. Engng. Rep. 333, 32 pp., Dec. 1955.

Results of rotor flutter tests performed by the NACA [AMR 8, Rev. 2471] are compared with corresponding values predicted by analysis. The rotor tested possessed blade pitch-control flexibility and flexible blades. The analysis which is performed for zero forward velocity of the rotor is of the conventional Rayleigh type using two degrees of freedom and accounts for the centrifugal forces. The aerodynamic forces are those given by strip theory using the basic results for a two-dimensional wing flying along a straight path.

Agreement between experiments and analysis is satisfactory. Reviewer remarks that this agreement will not be obtained under all circumstances when rotors are flutter-tested at zero forward speed. There exist cases of "wake" flutter which would be stable when the usual aerodynamic theory is employed. These cases can be understood when an aerodynamic theory is used which takes into account the fact that the blades are rotating in their own wakes, which will be published before long by Timman and reviewer. A. I. van de Vooren, Holland

4036. Molyneux, W. G., and Chapple, E. W., the aerodynamic effects of aspect ratio on flutter of unswept wings, *Aero. Res. Counc. Lond. Rep. Mem.* 2942, 12 pp., 1955.

If a set of semi-rigid rectangular wings, of equal chords but of different spans, are structurally dynamically similar, flutter analyses based on two-dimensional aerodynamics will result in identical results for each of the wings of the set. The aerodynamic aspect ratio of these wings, however, differs among the members of the set. There will be variations, therefore, among the actual flutter speeds of these wings. It is reasoned that if these variations are measured, they will give an indication of the isolated effect of changing aspect ratio.

Six different sets of wings were tested at low speeds. Results indicate that the affect of aspect ratio is given, approximately, by

$$V = V_0 \left(1 + \frac{0.78}{AR}\right)$$

where V is the flutter speed, V_0 is the calculated flutter speed (two-dimensional aerodynamics), and AR is the aspect ratio..

W. P. Targoff, USA

4037. Eckhaus, W., On the theory of oscillating airfoils of finite span in subsonic flow, *Nat. LucthLab. Amsterdam Rap.* F. 153, 34 pp., 1955.

Brief critical survey of title problem with primary emphasis on incompressible flow. J. W. Miles, USA

4038. Ijff, J., Bosschaart, C. A., and van de Vooren, A. I., Influence of compressibility on the flutter speed of a family of rectangular cantilever wings with aileron, *Nat. LuchtLab. Amsterdam Rap.* F. 147, 111–124, May 1954.

In this report, diagrams are presented showing, for a family of rectangular wing-aileron systems, the dimensionless flutter speed as function of the ratio between flexural and torsional resonance frequency.

The calculations have been performed for Mach numbers 0, 0.5 and 0.7. The relative mass parameter, the positions of the elastic and inertia axes, the aileron static balance, the control-cable stiffness, and the structural damping have been varied systematically.

For binary wing-aileron flutter, the allowed static unbalance of the aileron has been calculated.

From authors' summary

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 3834, 3851, 3996, 4017, 4029, 4138, 4148)

4039. Hartmann, M. J., and Wilcox, W. W., Problems encountered in the translation of compressor performance from one gas to another, ASME Semiann. Meet., Cleveland, O., June 1956. Pap. 56-SA-61, 12 pp. + 5 appendixes + 1 table + 11 figs.

By using gases such as Freon-12, tests of air compressors may be made at lower speeds, stresses, and powers, but flows must be dynamically similar if test results are to be applicable to air. Actually, complete dynamic similarity is seldom possible, but approximate similarity will often yield useful results. Paper gives equations to determine test conditions for best approximation to dynamic similarity and gives some data on effect of departures from this condition.

Although paper is useful contribution, reviewer believes that, since dissimilar variations of density during process are principal reason for lack of complete similarity, results which are just as valid can be obtained by less cumbersome methods.

C. W. Smith, USA

4040. Grabow, G., and Bock, K., Theoretical considerations on the motion of duplex steam pumps and its influence upon the pumping procedure (in German), *Maschinenbau-Technik*, **5**, 5, 253–259, May 1956.

4041. Bouwman, H. B., Specific size and speed of various flow machines (in Dutch), *Ingenieur* **68**, 9, W. 23–W. 28, Mar. 1956.

Simple dimensionless numbers are derived for specific size and specific speed of various flow machines. It is shown that, for a number of very different types of high-efficiency flow machines, there exists a close correlation between the specific impeller diameter and the specific speed. From author's summary

4042. Caini, V., An investigation on ejectors with a constant cross section mixing chamber (in Italian), *Aerotecnica*, **36**, 1, 20–23, Feb. 1956.

Determination of the values for the variables at the end of the mixing space is investigated for the case of a constant cross-section mixing chamber. One finds that the initial values cannot be completely arbitrary and the necessary condition of the mixing is given. The problem can have one or two solutions which differ by a shock compression.

The solutions of the problem are plotted as functions of the parameters; curves (having general validity) are also given for the entropy step in the shock compressions.

From author's summary

4043. Schlichting, H., Boundary losses in pump cascades (in German), *Z. Flugwiss.* **4**, 1/2, 35–40, Jan./Feb. 1956.

Study is based on measurements at Technische Hochschule, Braunschweig, reported by N. Scholz and K. Gersten, at Reynolds 4×10^5; Mach 0.12; blade-aspect-ratio 3.0; spacing ratio 1.0; blade angle 135°; inflow angles 138°, 144°, 150°, 156°. Boundary loss arises from interaction of plane-flow loss mechanism with secondary flow in layer on cascade lateral boundary. Comparisons are based on contours of total head loss over a cascade cell. Most restricted loss region, minimum total loss, maximum filament deflection, and peak pressure rise occur for inflow angle 144°. Loss region "kernel" is well within inflow boundary layer up to this angle with subsequent rapid inward progression and mid-blade stalling for "loss-kernel" thickness of "twice" inflow boundary layer. Actual outflow boundary layer thickness is same for all four inflow angles and about twice inflow value.

Boundary effects are eliminated by suction, for both pump and turbine cascade plane-flow comparison tests. Little boundary effect occurs on mid-blade flow and on pressure drop for turbine cascade. Roughly 50% reduction occurs in pressure rise and filament deflection from boundary effect on pump cascade. Stalling losses for pump cascade at high inflow angles with boundary effect eliminated is of same order as total losses. Separation into plane and boundary losses here is not enlightening.

Pump cascade measurements with boundary effect eliminated are compared with author's plane, frictionless calculations of AMR **8**, Rev. 3482, both for pressure rise and filament deflection; good agreement. Decelerated flow and pressure rise of pump cascade are more sensitive to boundary flow than for the accelerated flow and pressure drop in the turbine cascade. P. R. Hardesty, USA

4044. Staniforth, R., A corrected speed tachoscope, *Aero. Res. Counc. Lond. curr. Pap.* 224, 9 pp. + 6 figs., 1956.

It is often desirable in testing aerodynamic compressors to take all readings at fixed corrected speeds rather than true speeds. This corrected speed is defined as the actual shaft speed divided by the square root of the ratio of the absolute air-inlet temperature to the standard temperature (288 K). With the instrument described, this is possible with high accuracy (error < 0.1%) without further complication than the setting of a dial to the observed air-inlet temperature.

The latter operation could be dispensed with and the correction obtained directly from a temperature-sensitive device such as a thermistor or a resistance thermometer element.

From author's summary

4045. Ainley, D. G., and Mathieson, G. C. R., An examination of the flow and pressure losses in blade rows of axial-flow turbines, *Aero. Res. Counc. Lond. Rep. Mem.* 2891, 33 pp., 1955.

In the present report, much of the available information up to 1951 is studied and analyzed to determine magnitudes of gas pressure losses and deflections in a wide variety of blade rows, and also to determine the separate influences of blade shape and spacing, Mach and Reynolds numbers, incidence, etc. Effects of tip clearance are also studied.

Secondary losses are found to be large in many instances, especially in bladings with low reaction, a well-confirmed fact for several years. Another significant feature lies in the discrepancy between the tendency found by the authors for the optimum pitch-chord ratio to decrease at high gas outlet angles and the opposite prediction of Zweifel's and Howell-Carter's criteria.

In spite of a four-year publication delay, paper will still provide useful data and suggestions to jet gas-turbine designers.

P. Schwaar, France

4046. Jungclaus, G., Boundary-layer investigations in rotating channels with shear flow (in German), *Mitt. Max-Planck-Inst. Strömungsforschung* no. 11, 67 pp., 1955.

Very competent study of form-changing flow under constant volume, such as occurs in centrifugal compressor with straight blades, is based on theoretical investigations and supported by experimental results. Pohlhausen's approximation process is partially used in theoretical reasoning, while the experimental procedure is based upon Couette flow along smooth plates. Stipulating the similarity of such flow conditions in rotating as well as in stationary channels, author disputes present form of boundary equation in case of shear flow.

This work assists in understanding of some parameters influencing the compressor efficiency, due to velocity distribution with respect to growth of the boundary layer and its separation. Numerous photographs illustrate experimental documentation of the presented theory. *NACA Rep.* 954 (1950) is among the quoted references.

C. R. Bell, USA

Flow and Flight Test Techniques

(See also Revs. 3842,3853,4012,4013,4022,4030,4039,4043, 4082,4087,4099,4107,4162,4064,4065)

4047. Hertzberg, A., The application of the shock tube to the study of the problems of hypersonic flight, *Jet Propulsion* **26,** 7(part 1), 549–554, 568, July 1956.

Changes in composition of air occurring at high temperatures of hypersonic flight are briefly reviewed. Brief description is given of methods of investigating flow under these conditions, including various shock-tube techniques already described in other papers. Use of thin metallic films as resistance thermometers for heat-transfer measurements is described. Application of shock tube to study of chemical reaction rates at high temperatures is briefly discussed.

W. A. Mair, England

4048. Stine, H. A., and Winovich, W., Application of scattering theory to the measurement of turbulent density fluctuations by an optical method, *NACA TN* 3719, 21 pp., June 1956.

Theory of light-scattering in transparent turbulent gases is adapted to measurement of average integral scale and intensity of turbulent density fluctuations in such gases. Experimental method involves passing beam of parallel light through turbulent gas field and measuring light-scattering with telephotometer. Experimental photometric data taken through turbulent boundary layer in supersonic wind tunnel are found to be of form predicted by analysis.

Advantages are that optical method is more straightforward than hot-wire anemometry since light-scattering depends only on density fluctuations, and no probe or complicated electronic circuitry are required. However, only integrated values are obtained, and interpretation of experimental data depends on turbulence model employed.

T. P. Clark, USA

4049. Clevenson, S. A., Some wind-tunnel experiments on single-degree-of-freedom flutter of ailerons in the high subsonic speed range, *NACA TN* 3687, 18 pp. + 3 tables + 12 figs., June 1956.

Results of wind-tunnel tests of three wing models with various aileron configurations are presented. Density in the range of 0.08×10^{-2} to 0.58×10^{-2} slug per cubic foot has little effect on the initial amplitude or initial Mach number associated with the aileron oscillations (buzz). However, the frequencies decrease somewhat with decrease in density. The initial Mach number associated with buzz decreases with increasing angle of attack, whereas mass balancing and changes in spring stiffness in these tests have little effect. Increasing the aileron mass moment of inertia lowers the oscillation frequency. Placing the aileron at the wing tip delays the onset of buzz to a higher Mach number. There are experimental indications that the buzz range is limited to a range of Mach numbers above the wing critical Mach number. A comparison of the results of the test data with two previously published empirical analyses is made.

From author's summary

4050. Trouncer, J., and Moss, G. F., Low-speed wind-tunnel tests on a model of a jet tailless aircraft, *Aero. Res. Counc. Lond. Rep. Mem.* 2843, 36 pp., 1956.

Report gives the results of longitudinal and lateral stability tests made on a model of a jet tailless aircraft. It includes the effects of split flaps, trimming flaps, dive-recovery flaps, and four types of anti-tip-stalling device (slats, nose flaps, double split flaps, and letter-box slots). It also includes the effect of the ground in the landing condition.

From authors' summary

4051. Hartley, D. E., Spence, A., and Kirby, D. A., Model tests on the effects of slipstream on the flow at various tailplane positions on a four-engined aircraft. Part I. Tests with contra-rotating propellers, *Aero. Res. Counc. Lond. Rep. Mem.* 2747, 6 pp., 1955.

Systematic wind-tunnel tests have been made to investigate the effects of slipstream on the flow near the tailplane of a typical civil transport with four contra-rotating propellers. Tailplane height has been varied for each of several wing-body arrangements; only one tailplane and one propeller position have been used. This report presents the main results in the form of changes in mean downwash angle, and velocity at the tailplane, as functions of tailplane position, lift coefficient, and propeller thrust. It is shown that the regions of increased downwash and velocity each extend for a range of tailplane height of about one propeller diameter, while the region of increased downwash is displaced upwards a quarter of a diameter relative to the region of increased velocity.

A comparison of this work with flight results [*Rep. Mem.* no. 2701], in which the propellers were single rotating, shows an apparent difference in the spread of the slipstream between single rotating and contra-rotating propellers.

From authors' summary

4052. Kirby, D. A., Model tests on the effects of slipstream on the flow at various tailplane positions on a four-engined aircraft. Part II. Tests with single rotating propellers, *Aero. Res. Counc. Lond. Rep. Mem.* 2747, 30 pp., 1955.

This part of the report gives the results of tests which have been made on the model of part I [see preceding review] fitted with single rotating propellers.

a) As with contra-rotating propellers, the region of increased downwash due to slipstream is displaced upwards a quarter of a propeller diameter relative to the region of increased velocity.

b) Unlike the contra-rotating propellers, there is evidence of some spread in the slipstream proportional to the tail arm. The variation of downwash increment with incidence was less than that obtained with the contra-rotating propellers.

These conclusions imply that the destabilizing effect is smaller than for contra-rotating propellers. For the range of tail arm considered, the difference in slipstream spread would not have much

effect on the variation of the destabilizing effect with tailplane height. The difference in slipstream spread is not sufficient to explain differences from flight results [*Rep. Mem.* 2701].

From author's summary

4053. **Winter, K. G., Scott-Wilson, J. B., and Davies, F. V., Methods of determination and of fixing boundary layer transition on wind tunnel models at supersonic speeds,** *Aero. Res. Counc. Lond. Rep. Pap.* 212, 15 pp. + 14 figs., 1955.

An account is given of methods used in supersonic wind tunnels for observation of boundary-layer phenomena, in particular of the sublimation and oil-film techniques. Examples are given of the results of these techniques.

On the fixing of transition, a rough guide is given for the minimum size of wire required, with an example of the use of wires. The results of a brief experiment on the profile of an artificially promoted turbulent boundary layer are given.

From authors' summary

4054. **Gaukroger, D. R., Chapple, E. W., and Milln, A., Wind-tunnel flutter tests on a model delta wing under fixed and free root conditions,** *Aero. Res. Counc. Lond. Rep. Mem.* 2826, 10 pp., 1955.

Wind-tunnel tests on a model wing of delta planform are described. Tests have been made with the wing root fixed and also with the root free in pitch and vertical translation. Critical flutter speed and frequency are given for a wide range of variation of wing and fuselage inertia parameters. The results show that, under root-free conditions, body freedom flutter occurs at low values of fuselage pitching moment of inertia, but that at higher values the flutter is similar to that obtained with root-fixed conditions.

From authors' summary

4055. **Williams, J. L., Wind-tunnel investigation of effects of fuselage cross-sectional shape, fuselage bend, and vertical-tail size on directional characteristics of nonoverlap-type helicopter fuselage models without rotors,** *NACA TN* 3645, 39 pp., Mar. 1956.

Paper presents the results of a low-speed wind-tunnel investigation to determine the effect of fuselage cross section, fuselage bend, and vertical tail size on the directional stability characteristics of nonoverlap-type helicopter fuselage models.

From author's summary

Thermodynamics

(See also Revs. 3822, 3994, 4042, 4044, 4067, 4068, 4069, 4070, 4080, 4093)

4056. **Spencer, R. C., Meyer, C. A., and Baird, R. D., Properties of steam at high pressures,** *Mech. Engng.* **78,** 7, 615–619, July 1956.

An extension of the thermodynamics properties of steam from the values given in "Thermodynamic properties of steam," by Keenan and Keyes, at 5500 psia to 10,000 psia is made for temperatures up to 1600F. For temperatures above 960F, values are taken directly from the table by Hidalgo and Nutt of the United Aircraft Corporation. Between 640F and 960F, UAC values were rejected because they showed a double maximum for the specific heat along a line of constant pressure.

For temperatures less than 960F, the treatment consisted of a graphic development of constant-volume lines and constant-enthalpy lines on a pressure-temperature diagram. The constant-volume lines were anchored at the low temperature end, below 400F, to the data of Amagot. The isenthalpics were anchored at the high temperature end, 960F, to the UAC values. At intermediate temperatures, the procedure was rather arbitrary, although a single maximum for specific heat was insured at any one pressure. The properties were interrelated through the usual partial differential relations of thermodynamics which revealed inconsistencies which were "within the accuracy necessary for the usual engineering calculations."

According to authors, "sole purpose of this interim table is to provide a consistant and reasonable extrapolation of current tables that will be useful in power-system calculations until better data are available." J. H. Keenan, USA

4057. **Backstrom, M., Entropy-enthalpy diagram for water vapour and liquid extended to higher temperatures and pressures,** *Trans. roy. Inst. Technol. Stockholm* no. 104, 23 pp., 1956.

This steam diagram covers the range up to 800C and 2000 bars. Giorgi (MKSA) system of units is used throughout. The trend toward use of rational units in steam table is certainly welcome. The main basis for extrapolation up to 2000 bars came from two sources: theorem of corresponding states using data of various gases and volume data of Kennedy [*Amer. J. Sci.* **248,** 540–564, 1950].

Now it is well known that the reduced equation of states behaves quite differently for polar and nonpolar gases, so that it seems quite unlikely that the reduced properties of, say, hydrogen and Freon 13 could be used for computing the properties of steam. The use of Kennedy's data is indisputable, although it became known recently that the data are reliable only at not too high pressures, and numerical differentiation and integration in region where volume changes rapidly must be considered as very inaccurate from table values given only at wide steps. Values for moderate pressures (below 400 bars) are taken mainly from Faxén [AMR **7,** Rev. 3706] and from Moerkenstein and Schmidt [*Mitt. Ver. Grosskesselbesitzer,* no. 37, 714–723, 1955].

Chart is an oblique-angle Mollier diagram with wide meshes. Reviewer finds it rather inconvenient and inaccurate for practical use. Both liquid phase and vapor phase appear in the same diagram. L. S. Dzung, Switzerland

4058. **Jessup, R. S., A new Bunsen-type calorimeter,** *J. Res. nat. Bur. Stands.* **55,** 6, 317–322, Dec. 1955.

A new Bunsen-type calorimeter, using diphenyl ether as the calorimetric substance, is described. Tests of this calorimeter show that it can be used to measure quantities of heat of the order of 38 calories with a precision of about 0.05%.

From author's summary

4059. **Reilly, M. L., and Furukawa, G. T., Surface tension of oxygen, nitrogen, and their mixtures,** *Nat. Bur. Stands. Rep.* 3958, 39 pp., Feb. 1955.

The surface tension of liquid oxygen, nitrogen, and three of their mixtures was determined by the method of capillary rise in the temperature range from about 70 to 90 K. The results with oxygen and nitrogen are in fair agreement with those published previously by Baly and Donnan. The surface tension of the mixtures has been compared with an equation based on the ideal solution law and was found to agree with a maximum deviation of about 2½%. A slight modification of the equation using activity in place of mole fraction decreased the maximum deviation to about 2%.

From authors' summary

4060. **Carlson, W. O., and Schneider, P. J., Transport properties for binary gas mixtures,** *Univ. Minn. Inst. Technol. Mech. Engng. Dept. TR* 7, 32 pp. + 14 figs., Jan. 1956.

The diffusion process associated with two-component flow over flat plates has been found from recent studies to have a marked effect on the boundary-layer development. This effect is particularly pronounced when the properties of the diffusing foreign gas are widely different from the properties of the free stream. Since the thermal and dynamical properties affect the flow, there is a definite need for accurate information on the variation of these transport properties with both temperature and diffusing gas concentration.

A review has been made of the various methods available for calculating these properties, and methods have been developed

and suggested which lend themselves to rapid calculation without an excessive loss in accuracy.

Expressions for the following binary mixture properties are included: density, constant-pressure specific heat, thermal conductivity, binary diffusion coefficient, and thermal diffusion factor.

From authors' summary

4061. Anonymous, Thermal problems of atomic power stations, *Broun Boveri Rev.* **43**, 1/2, 7–15, Jan./Feb. 1956.

For the development and construction of atomic power stations the requirements of an efficient and economical thermal plant must be made to conform to the limitations imposed by the physical and technological requirements of a reactor. In this article thermal engineers express their views on these reciprocal effects and, as an example, demonstrate how interesting it is to perform the entire process of heat absorption as near as possible to the maximum temperature of the reactor. From summary

4062. Westley, R., Optimum design of a vortex tube for achieving large temperature drop ratios, *Coll. Aero. Cranfield Note* 30, 20 pp. + 32 figs., May 1955.

The vortex tube, invented by G. J. Ranque, is a simple device which separates a flow of compressed gas into a hot and a cold stream by means of a high-speed vortex. It may have useful applications as a refrigerator, and one of the aims of the present investigation is to widen the field of application by increasing the vortex tube's cooling performance.

The tests, described in this report, determined the effect of the hot valve setting, the cold outlet diameter, the inlet nozzle size and the inlet pressure ratios upon the temperature drop ratio characteristics of a vortex tube. The results show that, by matching the inlet nozzles and cold outlet diameter to the inlet pressure ratio, it is possible to obtain, over a wide pressure range, a temperature drop which is 0.50 of the isentropic temperature drop.

The optimum cold outlet area is almost independent of the inlet pressure, while the optimum inlet area decreases as the pressure ratio increases. From author's summary

Heat and Mass Transfer

(See also Revs. 3843,3873,3874,3877,3931,3979,4006, 4060,4061,4062,4101,4102,4116)

4063. Atalla, M. M., and Preston, K., Jr., Transient temperature rise in a semi-infinite solid due to a uniform disk source, *J. appl. Mech.* **32**, 2, 313–314 (Brief Notes), June 1956.

Heat conduction solution is presented for the temperature distribution in a semi-infinite solid due to a circular disk source of constant and uniform heat flux. The solution is shown to be related to the one-dimensional transient solution for the semi-infinite rod. The properties of the solid were considered invariant with temperature.

The solution was obtained as an answer to the problem of temperature distribution due to a plane surface anode under bombardment by a stream of electrons of uniform power density, but the results obviously have many other applications.

R. M. Drake, USA

4064. Fettis, H. E., Concerning the eigenvalues of a differential equation in convective heat transfer, *Quart. appl. Math.* **14**, 2, 198–200 (Notes), July 1956.

4065. Liboff, R. L., A useful integral formula for the initial reduction of the transport equation, *Quart. appl. Math.* **14**, 2, 200–201 (Notes), July 1956.

4066. Brisken, W. R., and Reque, N. J., Heat load calculations by thermal response, *Heat. Pip. Air Condit.* **28**, 7, 143–150, July 1956.

A new method of making heat load calculations is presented. The method starts out from electrical analog representation of the problem and makes it possible to solve even complicated problems without using analog computers. Instead of using artificial design conditions, this thermal response method punches business machine cards with actual weather data as recorded by the U.S. Weather Bureau; determines thermal response factors, and uses these factors to calculate loads by hand, by a desk calculator, or by a digital computer.

Samples are presented to demonstrate some of the diversified calculations possible with this method. A new application method based on thermal response is developed.

From authors' summary

4067. Krischer, O., and Esdorn, H., Simple short time method for the simultaneous determination of heat conductivity, heat capacity, and heat penetration of solids (in German), *VDI-Forschungsheft* **21**, 450, 28–39, 1955.

An unsteady method for measurement of specific heat c, heat conductivity k, and thermal diffusivity a is based on integral of Fourier's equation for unsteady, one-dimensional heat flow into a slab when, starting at a certain time, a constant heat flow enters through one surface (1) whereas the other surface (2) is adiabatic to heat flow. This relation shows that product $kc\rho$ (ρ density) can be calculated from increase in time of temperature of surface 1 during first phase of process when surface 2 does not experience any temperature variation. Heat conductivity k is obtained from difference of simultaneously measured temperatures in surfaces 1 and 2, product ρc from temperature gradient in time measured in surface 1, and product $k/\rho c$ from time interval occuring between passing of the same temperature in plane 1 and 2. The three last-mentioned values are obtained during later phase of process when temperatures in surface 1 and surface 2 increase linearly with time.

In experimental setup, conditions on which above relations are based are well approximated by guard-heating and proper choice of dimensions. Possible errors are calculated and found to be within 2%. Measurements on materials with known properties give accuracies of same order. One measurement takes 3 to 30 min. Method allows, therefore, measurement on moist materials.

E. R. G. Eckert, USA

4068. Krishnan, K. S., and Jain, S. C., Determination of thermal conductivities at high temperatures, *Brit. J. appl. Phys.* **5**, 12, 426–430, Dec. 1954.

Measurements of the thermal conductivity of platinum are given in the range 1200–1800 K. Measurements were obtained from wires heated electrically in a vacuum, where heat generated by electrical resistance is dissipated by radiation and conduction. The temperature at different positions on the wire is read by an optical pyrometer. Analytical solutions for temperature distribution near center of a short wire, which is parabolic, and away from center of a long wire, which is logarithmic, are given. Each solution contains the conductivity and can be solved for it for known values of temperature, emissivity of surface, electrical resistivity, and wire geometry.

Reported values for emissivity agree with previous data. No comparable data exist for conductivity, but low-temperature values are about the same as those reported here.

G. M. Ketchum, USA

4069. Newman, P. C., An investigation of the adiabatic calorimeter by the heat transfer analogue, *Fuel* **35**, 3, 295–302, July 1956.

An electric analog consisting of a network of conductances is used to simulate heat generation and flow in a cylindrical powdered-fuel calorimeter. Current is analogous to flow and obeys a finite difference approximation to heat-transfer equation. Heat generation is simulated by introducing currents at network mesh points. Assuming a heating law, voltage (temperature) measurements made with analog are used to determine if calorimeter temperature rise validly reflects the heat generation within simulated

fuel. Nonuniformity of temperature distribution in calorimeter is demonstrated under varying degrees of initial heating. Author concludes calorimeter cannot be considered adiabatic, and that errors in estimation of fuel heat generation may be serious unless temperature distribution is considered in heating calculations.

Author's work reemphasizes that a priori assumptions regarding temperature distributions can be pitfalls in engineering analysis.

R. B. Grant, USA

4070. Thomaes, G., Thermal diffusion near the critical solution point, *J. chem. Phys.* **25**, 1, 32–33, July 1956.

Thermal diffusion measurements have been carried out near the critical solution point by means of a new apparatus which will be described in another paper. The Soret coefficient D'/D for nitrobenzene + *n*-hexane shows a large increase in the vicinity of this point. This is a strong argument in favor of the existence of a thermodynamical factor in the isothermal diffusion coefficient as predicted by the thermodynamics of irreversible processes.

From author's summary

4071. Crawford, D. H., and McCauley, W. D., Investigation of the aerodynamic heat-transfer characteristics of a hemisphere-cylinder in the Langley 11-inch hypersonic tunnel at a Mach number of 6.8, *NACA TN* 3706, 38 pp., July 1956.

The objective was to extend the range of experimental data concerning local surface temperature, static pressure, heat-transfer coefficients, recovery factors, and boundary-layer conditions for hemisphere-cylinders. For this configuration in supersonic flow, the bow shock is detached and ahead of the nose, resulting in subsonic flow over a portion of the hemisphere surface. This results in less aerodynamic heating of the nose compared to cone cylinders or pointed circular arc ogives.

A series of four models were tested. A 3-in. O.D. × 10.5 in. long model containing 16 pressure taps was used to obtain pressure distributions. Surface temperature distributions were determined for a 3.025-in. O.D. × 10.5 in. long model and a 1.170-in. O.D. × 5.32 in. long model. Temperature recovery studies were made with a 2-in. O.D. × 5 in. long model.

Data were recorded for transient, nonisothermal surface temperature distributions. Heat-flow rates and heat-transfer coefficients were calculated by graphical integrations of the plotted temperature data. Equations were derived for transient heat flow to the hemisphere but not for the cylinder. The Reynolds numbers ranged from 109,000 to 1.03 million. The entire boundary layer over the models was shown to be laminar.

Within 45° of the stagnation point, heat-transfer coefficients could be correlated by a single Stanton number-Reynolds number relationship. Above 45°, the local Mach number became supersonic, and compressibility effects caused the data to deviate from Sibulkin's incompressible flow solution. All physical properties were evaluated just outside the boundary layer. E. Eckert [*WADC TR* 54-70] indicates that the simple subsonic heat-transfer relations may be used in supersonic flow if the physical properties are evaluated at the mean boundary-layer temperature proposed by Rubesin and Johnson [*Trans. ASME* **71**, 4, 385–388, 1949]. Reviewer suggests that the modified Sibulkin stagnation-point relation be applied to the entire hemisphere, using physical properties evaluated at the mean temperature proposed by Rubesin and Johnson. Also, the data for the cylinder should be restudied, utilizing this boundary-layer temperature concept.

Heat-transfer coefficients over the whole body were slightly less than those predicted by the Stine and Wanlass theory [*NACA TN* 3344; AMR **8**, Rev. 2527] for an isothermal equilibrium surface.

The pressure distribution data were found to be closely predicted by the modified Newtonian theory. Comparison with pressure data at other Mach numbers indicates the hypersonic similarity parameter law valid. Although pressure distribution model was of a different size and L/D ratio than the heat-transfer models, reviewer believes the data may be applied to all the models since the hypersonic similarity parameter of each was greater than 1.2.

Reviewer believes that the data analysis could have been simplified and improved. However, authors seem to have done an excellent piece of experimental work and have presented much needed data on an important problem.

R. J. Mindak, USA

4072. Galbiati, A., Heat exchange and friction losses in porous packed pipes (in Italian), *Termotecnica* **10**, 6, 241–249, June 1956.

In addition to a review of the technical knowledge on the subject, author gives two nomograms for the quick evaluation of friction losses and of heat-exchange coefficient of gases in pipes packed with small geometrical elements (spheres, cubes, hollow and solid cylinders) or with particles of known medium diameter and surface.

C. Codegone, Italy

4073. Oudart, A., Heat flux caused by pulsatory movement of viscous incompressible fluid (in French), *C. R. Acad. Sci. Paris* **242**, 26, 3038–3039, June 1956.

4074. Bory, C., Experimental study of forced convection in a moving wing (Favre Hyperconvection) (in French), *Publ. sci. tech. Min. Air, France*, no. 312, 70 pp., 1956.

An experimental study of the phenomenon of hyperconvection first considered by A. Favre (augmentation of the heat exchange between a solid obstacle and an air flow by providing the obstacle with a cover moving in the direction of the fluid flow). The moving cover used in the study was a portion of the surface of a rotating cylinder placed in a wind tunnel, the remainder of the cylinder being thermally insulated. The experiment shows that the heat exchange is much less influenced by the motion of the surface than the theory predicts. The phenomenon might, however, find some industrial applications, for, the relative velocity between wall and air being smaller, the friction is lessened without much variation in the convection coefficient. It may, therefore, reasonably be expected that certain thermal exchanges can be accomplished with less expenditure of energy.

A. E. Brun, France

4075. Martino, R. L., Heat transfer in slip flow, *Univ. Toronto Inst. Aerophys. Rep.* 35, ix + 97 pp. + 21 tables + 8 figs., Oct. 1955.

Author solves the Rayleigh problem of an infinite flat plate suddenly set in motion using the slip and temperature jump boundary conditions. Solution is then applied to finite plate by use of a linear transformation between time and distance. Results appear reasonable at large distances from leading edge but do not yield realistic results at the leading edge (the recovery factor tends to zero at the leading edge).

Reviewer believes the difficulty may be partly due to the linear transformation between time and distance, as this transformation neglects pressure gradients induced along plate due to presence of thick boundary layer. Also, as author points out, a sharp leading edge is in a free-molecule flow field which is neglected in this analysis.

Analysis does throw some light on the region near the start of the slip-flow regime.

G. Goodwin, USA

4076. Fainzil'ber, A. M., Solutions of similar vortex and temperature fields (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **100**, 2, 225–228, 1955.

Author is concerned with special case of like variation of velocity and temperature over streamlined bodies. Beginning with dynamic and energy equations for boundary layer in a viscous, perfect gas, author proceeds in an orderly fashion with various restrictions to differential equations for temperature and for vorticity in the boundary layer that are formally identical. Bibliography indicates author was unaware of western work of similar vein—probably because work reported here must have been done several years ago.

R. A. Burton, USA

4077. Han, L. S., Laminar heat transfer in rectangular tubes with combined free- and forced-convection, *J. Amer. Soc. nav. Engrs.* **67**, 1, 163–167, Feb. 1955.

Solution is obtained for fully developed flow with constant properties and a uniform axial temperature gradient. At each axial position, the wall temperature is taken as uniform around the cross section, which is an idealized assumption unless the tube walls are highly conductive in the peripheral direction. Examples approximating this type of flow configuration are encountered in nuclear reactor cooling applications and in forced flow cooling of rotating components with high centrifugal force fields.

The momentum and energy equations in two-dimensional form are combined into a fourth-order differential equation for the temperature distribution within the channel. The similarity of this equation to Lagrange's equation in the elastic theory of thin plates is noted, and a solution is obtained by series expansion. The final result gives the Nusselt number as a ratio of two double summations involving Rayleigh number and aspect ratio of the duct as parameters. For practical use it would be desirable to have plotted a family of curves of Nusselt versus Rayleigh number for various aspect ratios. It is interesting to note that the specific heat at constant volume c_v is utilized in the energy equation, as contrasted with the use of c_p by Hallman in a similar problem, AMR 9, Rev. 1980.

Some misprints in the paper are as follows: Eq. (8) should be a double summation; in Eq. (9), P should be ρ; in Eq. (10) n^2d^2 should be $n^2\beta^2$, n^4 should be inside the bracket, and the whole equation should be negative; in the expression for q above Eq. (11) an A should be deleted. For the final result, Eq. (12), to reduce to the results of reference 4 of the paper as stated, the double summation in the numerator should be inside the large bracket.

R. Siegel, USA

4078. Businger, J. A., Heat transfer in aerated packed beds of granular materials (in Dutch), *Ingenieur* **68**, 29, 0.87–0.90, July 1956.

The temperature in an aerated packed bed of onions is measured as a function of location and time. Results are compared with well-known theoretical equations. Generalized differential equations account for the effect of the presence of moisture. The drying time can be derived from a highly simplified model. Condensation and heating symptoms receive attention.

Reviewer remarks that the integral in equation (7) should be increased by unity. H. A. Vreedenberg, Holland

4079. Devienne, M., New method for the measurement of total emissivity coefficient and accommodation coefficient of a surface (in French), *C. R. Acad. Sci. Paris* **243**, 1, 27–28, July 1956.

Two identical plates facing each other are located in an evacuated isothermal enclosure. One of the plates is heated by a radiant energy source and the resulting temperature difference of the two surfaces is measured. To obviate the measurement of the net radiant energy input, the experiment is repeated after blackening the back face of the plate exposed to the radiant energy. As a result of these two experiments and from their respective appropriate relations for radiant energy exchange, one arrives simply to the relation $e = n'/(2n - n')$, where e is the total emissivity of the surface and n and n' are, respectively, the deflections of the galvanometer in the two experiments. The accommodation coefficient is obtained in a similar manner by radiating the blackened surface of the plate and again measuring the surface temperature difference of the two plates. From this experiment new simple relations are developed giving the accommodation coefficient of the surface.

Y. S. Touloukian, USA

4080. Min, T. C., Schutrum, L. F., Parmelee, C. V., and Vouris, J. D., Natural convection and radiation in a panel-heated room, *Heat. Pip. Air Condit.* **28**, 5, 153–160, May 1956.

Natural convection coefficients and radiation interchange factors based on the experiments made in the ASHAE Environment Laboratory under floor panel and ceiling panel heating conditions are reported. Convection values obtained in the test room for the room surfaces and for a small free-edge plate, and values for small plates as reported by other investigators, are compared.

Convection from the heated ceiling panel was found to be considerably less than from a small downward-facing free-edge plate. Convection coefficients determined for other surfaces of the room compared favorably with values determined from similarly oriented small heated plates.

Radiation exchanges between the heated panel and the room surfaces were measured with a radiometer and found to agree well with calculated values. From authors' summary

4081. Eckert, E. R. G., Hartnett, J. P., and Irvine, T. F., Jr., Measurement of total emissivity of porous materials in use for transpiration cooling, *Jet Propulsion* **26**, 4, 280–282, Apr. 1956.

Paper describes the construction and use of an apparatus for measuring the emissivity of surfaces and presents data for a sintered bronze material. In the range 160F–350F, the emissivity was determined to be 0.57 ± 0.04. At 350 F the sintered bronze oxidized rapidly, resulting in an increase in emissivity to 0.64. The specimen was ground to close the pores. For this newly formed essentially solid surface the measured emissivity was 0.04 at 150 F. W. M. Rohsenow, USA

4082. Maulard, J., Calibration equipment for pyrometers with sonic aspiration (in French), *Rech. aéro.* no. 49, 27–30, 1956.

4083. Optell, J. B., and Sage, B. H., Relations in material transport, *Indust. Engng. Chem.* **47**, 5, 918–923, May 1955.

A presentation of relationships among four diffusion coefficients defined in the literature. Limitations of the various coefficients are pointed out. Conditions are given where certain of the coefficients are identical in molecular transport processes.

Reviewer feels that principal engineering interest lies in portions of the article devoted to Fick's coefficient and to its relations with the other three. D. M. Vestal, Jr., USA

4084. Grossmann, U., Mass and heat transfer between liquid and rising steam bubbles in two phase mixtures (in German), *ETH, Zürich*, Prom. Nr. 2396, 60 pp., 1955; also *Chemie-Technik* **28**, 107–112, 1956.

Vapor was injected as bubbles beneath alcohol-water mixtures at several depths and photographed. Mass transfer was determined by analyses and heat transfer by bubble growth rates. Results are presented as ratio of concentration increase to initial concentration difference as a function of liquid depths up to 20 mm. No general correlation was obtained. Mass-transfer coefficients (12.5 to 24.2 kmol/sq m/hr/mol fraction) increased with increasing liquid concentrations, gas velocities, and decreasing nozzle size. The liquid superheat or subcooling, vapor superheat, and flush or pointed nozzles had only a minor effect. For alcohol-water, the heat-transfer coefficient averaged 31,000 Btu/hr/sq ft/°F and was independent of nozzle size, liquid concentration, or vapor velocity. For pure water, the coefficient was 14,500 Btu/hr/sq ft/°F. Correlations of von Krevelin and Hoftijzer on bubble diameters and volumes were confirmed. The rising velocity of the bubbles was greater than reported in literature.

This paper provides data necessary for a fundamental understanding of mass and heat-transfer phenomena in distillation and boiling operations. A. C. Mueller, USA

4085. Bankoff, S. B., Ebullition from solid surfaces in the absence of a pre-existing gaseous phase, 1956 Heat Transfer and Fluid Mechanics Institute, Stanford, Cal., Prepr. 1, 14 pp.

The nucleation theory of Volmer and Fisher is extended to the superheating of liquids in contact with various solid boundaries. It is shown that nucleation in the bulk phase, at flat surfaces, at sharp projections, or in wetted cavities can be dismissed from consideration as possible explanations for experimentally observed superheats. Unwetted cavities are preferred nucleation points; but the difficulties of filling these completely with liquid are so great that it is probable that nucleation almost always occurs at a

pre-existing gaseous phase in static stressing of the liquid. Nucleation in dynamic stress phenomena, such as ebullition, cavitation, or effervescence, always occurs at the boundaries of gas or vapor entrapped in surface cavities.

From author's summary by W. M. Rohsenow, USA

4086. Isakoff, S. E., Effect of an ultrasonic field on boiling heat transfer—exploratory investigation, 1956 Heat Transfer and Fluid Mechanics Institute, Stanford, Cal., Prepr. 2, 14 pp.

The presence of an ultrasonic field increased the maximum heat-transfer rate from an electrically heated platinum wire to boiling water by about 60% over the value obtained in the absence of such a field. Film boiling could also be made to revert to nucleate boiling by applying the ultrasonic vibrations. While the tests reported are by no means exhaustive, they show clearly that the application of vibration to a boiling fluid in the vicinity of a heat-transfer boundary profoundly influences the boiling phenomena. This suggests that ultrasonics might prove useful in carrying out related operations.

From author's summary by W. M. Rohsenow, USA

4087. Katarzhis, A. K., Kosterin, S. I., and Sheinin, B. I., Electrical method for detecting stratification of steam-water mixtures (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 2, 132–136, Feb. 1955.

Resistance of an electrically heated wire located in a pipe carrying vapor and liquid is used to distinguish which phase is in contact with the wire. Equations are given for time lag caused by thermal inertia of wire and by evaporation of an adhering liquid film. For a 0.05-mm platinum wire, thermal inertia causes a time lag of about 0.02 sec. Construction details are given for a probe used in studying flow of high-pressure steam-water mixtures. Using a 0.05-mm platinum wire, probe was found to function only as a resistance thermometer at heating currents below 0.74 amps. At higher currents, changes from a vapor to a liquid environment were detectable.

C. V. Sternling, USA

Book—4088. Latham, R. F., Naval boilers, 2nd rev. ed., Annapolis, Md., U. S. Naval Institute, 1956, viii + 111 pp.

Primarily written for basic instruction of U. S. Naval Academy midshipmen, work is a comprehensive textbook on naval boilers, feed water treatment, and maintenance and operation of boilers. It contains a brief history of development of boilers from 200 B.C. to modern naval systems and nuclear propulsion plants. Author describes details of all internal and external boiler components, including various types of naval boilers. Text is well illustrated with cuts and sketches of construction details. Good discussion of feed water treatment and the maintenance and operation of naval boilers is also given. Steam theory and boiler design principles receive only limited coverage.

Book is clearly written and is suitable for beginning students in marine engineering; it will be valuable reference for discussion of boiler types and construction details.

R. B. Couch, USA

4089. Nelson, E. T., An alternative approach to the design of 1-2 heat exchangers, *J. Inst. Petrol.* **42**, 338, 124–127, Apr. 1956.

Equations are derived for calculating the correct mean temperature difference for one-shell pass, two-tube pass exchanger for variable coefficient or areas. Equations are relatively simple, but require use of another equation to calculate the temperature between the passes. Reviewer finds no advantage of the equation over existing correction factor plots except author can handle case where coefficients are variable and pass areas different. The last equation for constant pass area and variable U can be further simplified to $(U \setminus t)_m = (U_1 \setminus t_2 - U_2 \setminus t_1)/\ln(U_1 \setminus t_2/U_2 \setminus t_1)$ where $\setminus t_1 = T_1 - (t_1 + t_2)/2$ and $\setminus t_2 = T_2 - t_c$.

A. C. Mueller, USA

4090. Franklin, J., and Keller, H. B., *A priori* bounds for temperature in circulating fuel reactors, *Quart. appl. Math.* **14**, 1, 57–62, Apr. 1956.

Paper is a mathematical treatment of the time and spatial dependent equations describing the neutron flux and temperature distribution in a circulating-fuel-type nuclear reactor. The equations dealt with are not derived in the paper. Authors discuss in detail the bounds of the flux and temperature within a circulating-fuel-type reactor for the case where the neutron production is a fairly general function of the local temperature.

R. S. Wick, USA

4091. Abel, G. C., Report of the third year's flying on the development of flight testing techniques for finding and measuring natural icing conditions, *Aero. Res. Counc. Lond. curr. Pap.* 223, 9 pp. + 4 tables + 8 figs., 1956.

This report covers the third year's search for natural icing conditions together with some results for the whole three-year period. Various forms of unserviceability limited this year's flying to approximately four months. For the three-year period only half the icing forecasts for layer-type cloud over U.K. yielded icing conditions sufficiently continuous to allow an anti-icing system to be tested. A number of these were too limited to allow more than one test run. By limiting testing to the best synoptic conditions yet found, forecasts will probably be completely correct although very limited in number. A good coating of ice was obtained from almost all forecasts for cumulus clouds. Occasions on which icing severity was overestimated during the past two years are double those in the first year, presumably due to the smaller number of upper air soundings available. The technique of photographing the leading edge of the tailplane has produced several interesting studies of ice formation. Due partly to the limited flying time, no great advance in instrumentation has been achieved this year.

From author's summary

4092. Prigogine, I., Non-linear problems in thermodynamics of irreversible processes, 1956 Heat Transfer and Fluid Mechanics Institute, Stanford, Cal., Prepr. 16, 18 pp.

A brief review of the thermodynamics of irreversible processes based on the Gibbs equation for the change of entropy of a phase and the reciprocal relations of Onsager-Casimir serves as an introduction to a treatment of nonlinear problems. The theorem of minimum entropy production for the stationary states is derived for the case of (1) linear phenomenological laws, (2) validity of Onsager's reciprocal relations, and (3) phenomenological constants L_{ij} may be treated as constants in subsequent variational process.

A more general treatment of time variation of entropy production in cases where some of the above restrictions do not apply involves the introduction of a function, analogous to the rate of entropy production in the restricted case, which is a minimum for the steady state. This function is a "velocity potential" in the case of two independent variables.

It is shown that the minimum rate of entropy production is not a general criterion of the steady state. Special criteria are developed for certain special cases. Spiral approach to and rotation about the steady state are discussed.

For the simple case of heat conduction it is shown that, if we prevent the system from reaching true equiblibrium for which W (heat flow) = 0, we obtain a stationary state for which the average value of W^2 is as small as possible. This condition corresponds to minimum entropy production if the temperature gradient is sufficiently small. The most striking conclusion "is that every stationary nonequilibrium state is characterized by the minimum of some quadratic function in the rates of the irreversible processes which has a general thermodynamic significance."

In the opinion of the reviewer, the introduction of the first-law equation (4.9) requires further justification.

J. H. Keenan, USA

Combustion

(See also Revs. 4058, 4069, 4110)

4093. Powell, H. N., Applications of an enthalpy-fuel/air ratio diagram to "first law" combustion problems, ASME Semiann. Meet., Cleveland, O., June 1956. Pap. 56-SA-68, 26 pp. + 4 figs. + 9 tables.

Author describes a simple technique for solving combustion problems involving the first law of thermodynamics, such as determination of adiabatic flame temperatures, theoretical heat releases, etc. This involves the use of "absolute" enthalpies, which are enthalpies calculated with respect to the elements in their standard states. Tables of absolute enthalpy and heat capacity for hydrocarbon-air systems are given.

T. L. Cottrell, Scotland

4094. Kimura, I., and Kumagai, S., Spark ignition of flowing gases, *J. phys. Soc. Japan* **11**, 5, 599-604, May 1956.

Spark ignition of flowing gases is investigated experimentally at room temperature and atmospheric pressure. Under laminar condition, the inflammability of flowing gases increases with gas velocity. This is due to the tendency of initial flames to escape from the quenching effect of electrodes under the influence of gas flow. Under turbulent condition, the inflammability of flowing gases decreases with increasing gas velocity. This is ascribed to the increased eddy diffusivity and is in accordance with the result of Swett's experiment under turbulent condition at reduced pressures. From authors' summary

4095. Combustion in turbulent flow, Proceedings of a symposium at the Applied Physics Laboratory, *Jet Propulsion* **26**, 6, 480-495, June 1956.

In 16 pages of summaries and abstracts *Jet Propulsion* presents essence of material discussed at APL symposium of April 1955. The material, highly pertinent in spite of aging, includes papers on experimental techniques, theoretical concepts, and new investigations of practical applications of turbulent flames, as well as two papers on the characteristics of laminar flames.

Meeting brought together chief protagonists of opposing concepts of turbulent flame structure, concepts being the wrinkled laminar flame (Karlovitz) and the reaction-in-depth (Summerfield). Each concedes that the other's theory probably holds for some flames at some condition of turbulence but there appears to be no agreement on the limits of applicability of either flame model. Seven other authors on the same panel took views intermediate between those of Summerfield and Karlovitz; some questioned the validity of experimental evidence used to support one or the other theory and one, at least, (Weiss) contended that neither concept is useful in describing flames spreading from baffles.

Several workers described experimental measurement of turbulent burning velocities by the technique of photographically following growth of a flame kernel in a turbulent stream. A new technique for studying the characteristics of turbulent flames was use of helium diffusion, described by Westenberg.

Net results of the meeting were to expose and clarify differences of opinion, to point up some new ideas, and to emphasize need for more precise techniques for measuring point conditions within turbulent flames. W. P. Jensen, USA

4096. Miesse, C. C., On the stability of flame fronts in the combustion of liquid fuel droplets, Amer. Rock. Soc. Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 254-55, 20 pp. + 4 figs.

The stability of flame front in the combustion of liquid fuel droplets is investigated theoretically by assuming that (a) each droplet constitutes a spherical vapor source, (b) the flame front is the locus of all points at which the stoichiometric condition is satisfied, and (c) the burning rate and variation of droplet radius are determined by published theoretical and experimental equations. Consideration of the time-lags inherent in the phenomenon leads to an oscillatory solution with exponential growth of decay, which can be expressed as a small perturbation added to the non oscillatory solution obtained by the omission of the time-lag considerations. For the monopropellant case, the frequency can be determined directly if the time-lag is known, and the stability criterion is expressed in terms of the stoichiometric mixture ratio and an explicit function of the physical properties of the droplets and its vapor. For the bipropellant case the predicted frequency of oscillation can be determined only by numerical solution of a transcendent equation, but the stability criterion is expressed simply as the solution of a quadratic equation which involves the physical properties of the droplets and the stoichiometric mixture ratio. As a result of this investigation, it can be shown that the oscillation under consideration can be reduced by reducing the size of the droplets, or (in the bipropellant case) by choosing the propellants so that an explicit function of the physical properties and stoichiometric mixture ratio is equal to unity. Theoretical analysis; Zero-order Solution; First-order Solution; Single-propellant case; Mass-flow of oxidizing Vapor; Constant oxidizer flow-rate. Conclusions: (1) oscillations of any frequency can be established by the instability of flame front surrounding a fuel droplet in either bipropellant or single-propellant combustion, (2) the frequency increases with an increase in the size of the fuel droplet (in either system) and with the decrease in the distance between droplets in a bipropellant system, (3) stability is increased as the size of the droplet decreases in the bipropellant system and in the constant oxidizer flow-rate single-propellant system.

K. J. De Juhasz, Germany

4097. Neimark, Yu. I., and Aronovich, G. V., The conditions for self-excitation of a singing flame, *Soviet Physics (JETP)* **1**, 3, 475-483, Nov. 1955 (translated from Russian, *Zh. eksp. teor. Fiz.* **28**, 567-578, May. 1955, by R. T. Beyer).

A Satche-Nyquist type of analysis is used on three successively more complex models of a singing-flame apparatus; namely, (a) a Helmholtz supply system and Helmholtz combustor, (b) a tubular supply system and a Helmholtz combustor, and (c) a tubular supply system, a cylindrical combustor, and a flame of finite size. The usual time lag term is also introduced. Only the predictions based on the third model agreed with experimental results reported in the literature in all details. These predictions are clearly stated, but are only qualitative in nature. A. A. Putnam, USA

4098. Klein, G., Equations of a simple flame solved by successive approximations to the solution of an integral equation. Part IV. Simple ideal flame model suggested by the HBr flame, *Univ. Wisc. Nav. Res. Lab.* WIS-SQUID-3, 98-113, Apr. 1955.

The methods of parts I-III [AMR **8**, Revs. 1517, 1518, 3350] have been adapted to an ideal flame based on the reaction $A + B \rightarrow C$, where the masses and mutual diffusion coefficients in the three-component system are unequal. In this article, the afore-mentioned methods are applied to the HBr flame. In present treatment, the free radicals H and Br, which are responsible for the chemical reaction (e.g., chain reactions), enter into the formulation only indirectly; concerning diffusion of Br_2, H_2, and HBr only, burning speed is calculated by solving the flame equations using successive approximations (cf. parts I-III). C. Franze, Germany

4099. Weinberg, F. J., The shadowgraph of a flat flame, *Proc. roy. Soc. Lond.* (A) **235**, 1203, 510-517, June 1956.

Author attempts to correlate the shadowgraph intensity distribution with flame structure. The structure of the parallel light shadow of a flat flame is described. For a shadowgraph close to the flame, temperatures corresponding to maximum and minimum light intensity are calculated. For air, these temperatures are 1.14 and 2.19 times the initial temperature, respectively. For a shadowgraph far from the flame, author proposes the line of the first discontinuous increase in intensity as a possible point of reference. For air, this line corresponds to a temperature 1.5 times the initial temperature. The minimum distance between flame and screen for easy interpretation of this reference line was calculated and is reasonable for experimental purposes. A. C. Scurlock, USA

4100. Fine, B. D., and Wagner, P., Space heating rates for some premixed turbulent propane-air flames, *NACA TN* 3277, 26 pp., June 1956.

Space heating rate (SHR) is defined as product of space conversion rate (volume flow rate /volume of flame brush) and quantity of heat released by unit volume of cold reaction mixture on passing through flame zone. SHR measurements can yield information on flame thickness and chemical reaction in flame zone which is useful in characterizing turbulent combustion. Wide range of flow conditions is studied; also the effect of burner diameter.

For Bunsen burner flames stabilized over a field of pipe-induced turbulence, SHR decreased with increasing linear flow and burner diameter and was independent of pilot conditions. Values were in range 2×10^7 to 6×10^7 Btu/cu ft/hr. (These are an order of magnitude lower than for laminar flames). Turbulent burning velocities over same flow range were correlated by linear velocity at constant burner diameter but variation with burner diameter was complex.

Method is limited by lack of definition of turbulent flame brush boundaries. M. F. Hoare, England

4101. Putnam, A. A., and Dennis, W. R., Pulsations in residential heating equipment; preliminary results, *Heat. Pip. Air Condit.* 28, 7, 131–138, July 1956.

Progress of an investigation of pulsations in oil-fired and gas-fired heating equipment is described. The research program has the objectives of defining the fundamental nature of the pulsations and of finding methods for their suppression or complete elimination. In an initial field survey, it was found that each unit tested generated standing waves of a type similar to those produced in a doublenecked resonator. Conventional furnaces were then studied briefly in the laboratory, and the results obtained were applied to the design of two experimental furnaces.

Two experimental techniques have been used: (1) motion-picture studies, and (2) tests in which important factors, such as air-fuel ratio, are varied and their effect on oscillations noted. From the preliminary results presented in this report, the following can be stated: (1) For the oil-fired unit, the important factors in oscillations appear to be: (*a*) the relative flow patterns of fresh air, hot products of combustion and fuel, and (*b*) the relative positions of the combustion zone and the pressure antinode. (2) For the gas-fired unit, the most important variable appears to be the rate of flow of primary air. (3) Motion-picture studies indicate that the mechanisms for driving of oscillations are related to periodic, rapid flame expansions for the oil-fired unit, and a vortex shedding, for the gas-fired unit. From authors' summary

4102. Lees, B., An investigation into the air-heater corrosion of oil-fired boilers, *Combustion* 28, 1, 38–42, July 1956.

Severe air-heater corrosion has been experienced in the oil-fired boilers at Bankside Station, Central Electricity Authority. Methods which have been successfully applied to reduce the fouling and wastage are described. Future experiments to ascertain optimum operating conditions, and suitable air-heater design are also considered. From author's summary

Acoustics

(See also Revs. 3857, 3913, 3962, 4003, 4086)

4103. Botta, A., A simple calculation method for the absorption of sound due to viscosity in narrow channels (in French), *Bull. tech. Suisse Rom.* 82, 11, 170–173, May 1956.

This paper considers the problem of sound absorption due to viscosity alone, in the case of plane wave propagation along narrow channels. Only one-dimensional problems are considered, between parallel planes, and in a straight tube whose cross section is a circle. In the course of the work, author makes approximations without adequate explanation. It would appear that the results do not represent any improvement on the classical results of Rayleigh. J. M. Jackson, Scotland

4104. Kraichnan, R. H., Pressure fluctuations in turbulent flow over a flat plate, *J. acoust. Soc. Amer.* 28, 3, 378–390, May 1956.

Pressure fluctuations exerted on a flat plate by a turbulent boundary-layer flow are investigated. Approximate dependence of mean-square intensity, spatial scale, and frequency scale on Mach number and distance from transition point are estimated through use of similarity arguments. Wave-number spectrum of pressure fluctuation distribution over surface of plate is expressed in terms of transforms of two-point velocity correlations, and expressions are derived for driving force exerted on rectangular piston set in surface of plate. It is found that integral over boundary surface of two-point quadratic correlation function of pressure fluctuation should vanish, with result that mean-square force per unit area exerted on a large area of surface should tend to zero as area increases indefinitely.

Idealized model of turbulent boundary-layer flow is constructed and used to relate spectrum and correlation function of surface pressure distribution to corresponding functions for homogeneous turbulent flow. Application of theory to Laufer's data on turbulent channel flow indicates that dominant contribution to pressure fluctuations should depend only on mean velocity profile and two-point quadratic correlation of fluctuating velocity component perpendicular to the boundary surface. On basis of seriously incomplete data available, the rms pressure fluctuation from this source is tentatively estimated.

Treatment of present paper is based on assumptions of (1) adiabatic conditions, (2) small compressibility, and (3) slow rate of growth of boundary layer.

From author's summary by H. S. Ribner, Canada

4105. Pryor, A. W., The propagation of sound in solutions of rubber and perspex, *Acustica* 4, 6, 658–661, 1954.

4106. Waller, Mary D., Critical review of earlier Chladni figures, *Acustica* 4, 6, 677–680, 1954.

4107. Beranek, L. L., Labate, S., and Ingard, U., Noise control for NACA supersonic wind tunnel, *J. acoust. Soc. Amer.* 27, 1, 88–98, Jan. 1955.

To reduce an intense noise with frequency components extending from 5 to 10,000 cps due to the burning of a jet engine in the test section of a supersonic wind tunnel, a special muffler was built having an open cross-sectional area of 600 sq ft. For frequencies below 11 cps, large Helmholtz-type resonators were used. Between 11 and 20-cps, intermediate-sized Helmholtz resonators were employed. Between 20 and 800 cps, a tuned set of six Fiberglas lined square ducts were constructed. At higher frequencies, two lined bends were employed to reduce the noise. Performance data measured in models and in the full-scale wind tunnel are presented. The result is a wind tunnel that is so quiet that nearby listeners are unaware of its operation.

From authors' summary

4108. Lange, Th., The natural frequency of horns (in German), *Acustica* 5, 6, 323–330, 1955.

The impedances at the throat and natural frequencies are calculated for horns which consist of a pressure chamber, a connecting tube of constant cross section, and a terminating SALMON horn (widening in the hyperbolic or catenoidal form). In spite of uncertain end-corrections the calculations agree with more precise measurements. The resonances are inharmonic, because of the pressure chamber as well as of the widening of the horn. In special cases it is possible to offset the two influences so as to produce natural frequencies in harmonic relationship.

From author's summary

Ballistics, Detonics (Explosions)

4109. Hunter, M. W., Shef, A., and Black, D. V., Some recent aerodynamic techniques in design of fin-stabilized free-flight missiles for minimum dispersion, *Douglas Aircr. Co. Rep.* SM-19532, 29 pp., Jan. 1955.

Paper, an extension of work in AMR 1, Rev. 912, derives techniques for reducing dispersion of free-flight rockets due to "angular errors" introduced during thrust phase.

Establishing "yaw-wave-length" as index, authors optimize vehicle dispersion by utilizing closed-form transfer function solutions of equation of motion derived under various disturbances. Combining functions for thrust malalignment dispersion and dispersion due to error in wind assessment, paper presents equations for optimum yaw-wave-length and minimum latent angular dispersion, assuming short launcher length. Factors in equation for minimum dispersion are analyzed to reduce level of dispersion. Factors designated as internal errors are shown amenable to control by spin techniques. Analysis for constant spin rates and launcher lengths indicates spin is most effective at very beginning of flight, wasted later. Authors feel factors designated external errors, mainly wind-velocity-assessment error, are dictated by glide-phase stability requirements; thus they recommend increased accuracy of wind measurement. As in spin study, wind-velocity-assessment error shows largest effect at beginning of flight, allowing degrading of measurement accuracy with distance from launcher.

Rocket admittance theory of Robinson [*Douglas Rep.* SM18443, Jan. 1955] is used to demonstrate importance of design of rocket-borne wind-sensing instruments. Extension of present analysis requires additional statistical wind information. Even without such additional information, authors contend admittance function for matching rocket and instrument dynamic response characteristics can improve choice of wind-measuring equipment, leading to reduction of dispersion due to wind-assessment errors.

Reviewer believes paper is important logical extension of referenced work that can serve as stepping stone for rocket system designer. H. L. Bloom, USA

4110. Pettey, T. M., Jr., Ground handling and launching procedures for an alcohol-oxygen rocket missile, *Jet Propulsion* **26,** 6, 474–479, June 1956.

Soil Mechanics, Seepage

(See also Revs. 3955, 4124, 4125, 4129, 4131)

Book—4111. Hammond, R., Foundation engineering, New York, Philosophical Library, Inc., 1956, 192 pp. $10.

Prospectus on dust jacket of book states: "Dealing exclusively with foundation problems and their solution, this volume covers a wider field than any other work of similar size published either in Great Britain or America and is believed to be the most up-to-date and concise book of its kind. —Although the book is of great value to the young civil engineer and student, it offers the result of a vast experience to the practising engineer ...." Scope of book may be seen from chapter headings as follows: Soil mechanics; Site exploration; Piled foundation; Vibration-controlled foundations; Foundations for houses; Foundations for buildings; Foundations for bridges; Foundations for maritime structures; and Difficult foundation problems.

Author undoubtedly has wide knowledge and experience in the field, as evidenced by numerous excellent examples of foundation practices. However, the reader will not find information on foundation design procedures and analytical methods; these are sadly lacking. The implication throughout the book is that the engineer learns by practice alone.

Chapters on soil mechanics and foundation exploration by no means cover adequately, nor do they properly emphasize, what is known in these fields and their relation to foundation engineering. The young engineer would be led astray without other knowledge or experience to guide him. Author pays great attention to details as to how various devices or instruments work, but provides little or no information as to how results may be used in design.

Book is poorly organized in that information following center headings often does not coincide with the subject being discussed. Although the author draws on world-wide illustrative examples, he has taken little advantage of knowledge and practice beyond that of his own countrymen. For example, of 56 literature references cited only five were from outside Great Britain. In summary, the book has numerous illustrative examples of foundation practice but does not constitute a text on foundation engineering.

W. G. Shockley, USA

4112. Nishida, Y., A brief note on compression index of soil, *Proc. Amer. Soc. civ. Engrs.* **82,** SM3 (*J. Soil Mech. Found. Div.*), Pap. 1027, 15 pp., July 1956.

This paper reports a new relationship between the compression index and the void ratio of soils after a theoretical consideration from some simple assumptions.

The relationship given by the author agrees well with experimental results of verification and gives a new approximate method to estimate the compression index of soils as a linear function of the void ratio. From author's summary

4113. Drucker, D. C., Gibson, R. E., and Henkel, D. J., Soil mechanics and work-hardening theories of plasticity, *Proc. Amer. Soc. civ. Engrs.* **81,** Pap. 798, 6 pp. + 11 figs., Sept. 1955.

The Coulomb criterion is not suitable for a soil that undergoes plastic volume change under an equal all-round effective pressure. A yield cone with a spherical cap on the open end of the cone is suggested as the yield surface. The soil is treated as a work-hardening material and this is taken into account by changing the size of the cone.

The discussion of Fig. 8b is interesting. It shows the unloading paths for which a soil that was hydrostatically loaded will undergo a plastic volume decrease or increase. For a particular unloading path no volume change occurs. Experimental data are given in support of the suggested yield surfaces.

E. D'Appolonia, USA

4114. Getzlaff, G., Forces in mouldboard and disk ploughing (in German), *ZVDI* **98,** 5, 172–175, Feb. 1956.

Extensive measurements made in the Institute for Basic Agrotechnical Research of the Agricultural Research Station in Braunschweig showed the influence of the following parameters on the forces in mouldboard ploughing: furrow width and depth, soil resistance, plough shape and size, changes in the inclination of the plough body with the axes, ploughing speed, breaking up of the furrow slice (effect of skim coulter), truncation of the body, peak forces in working stony soils. (A hydraulic six-component dynamograph and a modified Ott-Integraph were used.) Some remarkable partial results are discussed. Similar measurements were also started on disk ploughs with normal and with power-driven disk body, the brief initial results showing no advantages of the power-driven type.

From author's summary by S. O. Fejer, New Zealand

4115. Rocha, M., Folque, J., and de Castro, G., Compaction of earth dams—some results of field and laboratory tests; its correlation with fundamental themes (in Portuguese), *Lab. nacional Engen. civ. Lisboa Pub.* 59, 19 pp., 1954.

This publication presents some results of the studies and field tests made before and during the construction of the Silves dam, composed of a central core of clay and two lateral parts of weathered schist. The whole program of tests is divided in: (1) Field tests; (2) comparative study between field and laboratory tests; (3) study of the dispersion of the properties of the materials incorporated into the dam, using statistical methods as much as possible; (4) observation of the behavior of the dam under service, particularly displacements and total and neutral stresses. The possibility of studying these problems by means of models is considered.

From authors' summary by A. J. L. Bolognesi, Argentina

4116. Bakulin, F. G., and Zhukov, V. F., Deformation of frozen dispersed soils while thawing (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 7, 132–136, July 1955.

While freezing, the mineral aggregates between ice lenses are compressed up to the density corresponding to a pressure of 6–8 kgs/cm². Thus when thawing, the volume of soil loaded merely by dead load decreases due to partial rupture of the spongy macrovoids structure only. For a complete elimination of macrovoids, additional load up to 6–8 kgs/cm² is necessary. The swelling of the compacted mineral aggregate due to the influence of the melted ice can reduce considerably the decrease of the total volume.

Authors believe that there are various structures of frozen soils, depending on the degree of dispersity, on moisture content, on speed of the cooling down, etc. Because of the very heterogeneous character of such soils it is impossible to conclude their deformations due to thawing on the basis of tests of some small samples only. The structure of the frozen soil as a whole is to be investigated. L. Šuklje, Yugoslavia

4117. Galli, L., Soil-mechanics and hydrologic lessons drawn from the flood (in Hungarian, with English summary), *Viz. Közl.* 37, 157–171, 1955.

The safety of levees against floods depends on soil stratification. The subsoil in the Danube valley is composed of highly pervious sandy gravel covered by 1 to 6 m of silt or clay; the river channel is cut into this pervious layer. Different seepage ways and their action are discussed. A certain design width exists beyond levees, a limit for slides or scours. Measures are suggested to reduce hydrostatic pressure. S. Kolupaila, USA

4118 Terzaghi, K., Fifty years of subsoil exploration, Harvard Univ., Soil Mechs. Series no. 47, Cambridge, Mass., 227–237, 1953.

4119. Glushkov, G. I., Determination of horizontal stresses in the soil (in Russian), *Gidrotekh. Strout.* **23,** 3, 39–40, 1954.

Author worked into a graph the Boussinesq formula for the horizontal stress in an elastic semi-infinite solid caused by a concentrated vertical load. The use of this graph is of special advantage when computing the pressure at rest on a retaining wall caused by a multiform loading, as is demonstrated on an example. Considering that the wall does not take any deformation, it is necessary to multiply the result by 2, as is generally done when computing the pressure at rest. Author also mentions the use of this method for walls which deform, using a coefficient reduced from 2 down to 1. This is not quite correct, because it is then a case of active pressure. The graph is useful for quick computations. V. Mencl, Czechoslovakia

4120. Berezantsev, V. G., Limit loads in indentation of cohesive soils by spherical and conical indentors (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 7, 70–74, July 1955.

Paper gives the relationship between the limit loads in indentation and the friction and cohesion constants in a medium whose yielding condition is described by Mohr's law. Author proposes to use this relationship for the determination of these two physical constants in a cohesion soil.

The method employed is similar to that of Ishlinky's analysis of Brinell tests. It involves stepwise integration of the slip-line field obtained through use of Haar-Karman's assumption that two of the principle stresses are numerically equal. The validity of the results are, therefore, limited by this assumption.

The ratios of indentation load to cohesive strength of the soil are numerically computed for various friction angles, cone angles, and indentation depths. K. N. Tong, USA

4121. Meyerhof, G. G., Penetration tests and bearing capacity of cohesionless soils, *Proc. Amer. Soc. civ. Engrs.* **81,** SM1 (*J. Soil Mech. Found. Div.*), Pap. 866, 19 pp., Jan. 1956.

Relationships between dynamic and static penetration tests and bearing capacity of friction piles and spread footings are developed on the basis of field data. Results are not too reliable for skin-friction estimates in general, but are quite reliable for specific cases; i.e., H pile. Correlations are also made between relative density and angle of internal friction and penetration tests for cohesionless soil.

Reviewer believes the data presented demonstrate the limitations which must be observed when using penetrometers for making estimates of soil strength. J. A. Cheney, USA

4122. Handy, R. L., and Davidson, S. T., Rubber-balloon apparatus for measuring densities of soils in place, *Highway Res. Board Bull.* **122,** 13–22, Jan. 1955.

4123. Horonjeff, R., and Javete, D. F., Neutron and gamma-ray methods for measuring moisture content and density to control field compaction, *Highway Res. Board Bull.* **122,** 23–34, Jan. 1955.

Micromeritics

4124. Wooddy, L. D., Jr., and Moscrip, R., Performance calculations for combination drive reservoirs, *J. Petr. Technol.* **8,** 6, 128–135, June 1956.

Paper includes solutions of volumetric balance, unsteady state, and displacement equations for determination of oil recovery from reservoirs. Method of solution is capable of providing information on future performance of reservoirs under different modes of pressure application. Paper includes excellent analysis of saturation-distribution profiles for initial and subsequent periods of production. J. M. DallaValle, USA

4125. Geertsma, J., Croes, G. A., and Schwarz, N., Theory of dimensionally scaled models of petroleum reservoirs, *J. Petr. Technol.* **8,** 6, 118–127, June 1956.

Authors present an extremely useful application of dimensional analysis, thus facilitating understanding of many complex reservoir problems. Significance of various dimensionless groups is fully discussed. Conditions under which a reservoir may be dimensionally scaled are set forth. Paper is an important contribution to understanding reservoir behavior.

J. M. DallaValle, USA

4126. Kovacs, G., Some hydraulic aspects of seepage control (in Hungarian), *Hidrológiai Közlöny* **36,** 2, 113–123, Apr. 1956.

The amount of water percolating through a pervious layer under a levee and to be drained off through channels on the protected side of the levee can be reduced by sheet piling under the levee or by an impervious apron in front of it. Assuming two-dimensional potential flow, the best position for the sheet piling is shown to be along the centerline of the levee, and the apron length giving the same protection as a certain depth of piling is calculated. The size of the drainage channel required to collect safely the seepage for various conditions is determined and the protection (berm) to be supplied to guard against the damage at the toe of the levee due to subsurface erosion is calculated.

The calculations (illustrated by an example) give a sound basis to compare the economics of various solutions giving equivalent hydraulic protection. G. Sved, S. Australia

4127. Taylor, G., Fluid flow in regions bounded by porous surfaces, *Proc. roy. Soc. Lond.* (A) **234,** 1199, 456–475, Mar. 1956.

Incompressible fluid passes through a porous cone-shaped wall with vertex semi-angle α. In *outflow*, the assumption of uniform parallel *flow within the conical space* is legitimate; in *inflow*, however, the motion must be accelerated, and it is analyzed under the following assumptions: (1) The pressure drop across the porous wall is proportional to the square of the normal component of the velocity (resistance coefficient c); (2) α is small; (3) no mixing occurs (streamline flow); (4) the speed of the fluid leaving the wall can be neglected in Bernoulli's equation; (5) the pressure gradient normal to the cone axis is negligibly small.

Under these conditions, mass conservation and Bernoulli's equation lead to an integral equation for the pressure along a streamline considered as a function of the length of the axial projection

of the streamline, the solution of which is a power law with an exponent depending only on $\alpha c^{1/2}$, the governing parameter of the problem. Application of the same basic principles yields also the speed distribution in a transversal plane (table and diagram for various values of $\alpha c^{1/2}$). The problem of inflow through a wedge-shaped wall is also solved. The inflow through a *cylindrical* wall is a limiting case ($\alpha \to 0$) of the cone.

When (1) is relaxed the integral equation can still be solved explicitly for the cylinder, if the pressure drop is proportional to a power of the velocity higher than the second. When (3) is replaced by assuming complete mixing (constant flow speed on a transversal plane), but (1) is maintained, then mass and momentum conservation again lead to a solution, a power law for the pressure-distance and for the speed-distance relations. In this case the discharge coefficient is somewhat greater than in case (3), the difference decreasing with increasing $\alpha c^{1/2}$.

This theory is checked by measuring the discharge coefficient and the transversal velocity distribution of a jet emerging vertically from a cone or cylinder placed above an orifice in the bottom of a water tank. The porous material, whose resistance coefficient is determined independently, is perforated metal sheet and/or wire gauze. The agreement with the theory is very satisfactory.

The relation of the subject to problems arising in boundary-layer suction and to draining processes in paper manufacturing is briefly mentioned; the quantitative remarks on the resistance of porous sheets are of general interest. G. Kuerti, USA

4128. Rammler, E., On the laws of particle distribution of comminuted materials (in German), *Forschungen u. Fortschritte* **30**, 1, 1–9, Jan. 1956.

Importance and significance of particle-distribution curve and formula for describing the main characteristics of comminuted materials are pointed out. Survey of theoretical work in this field is given, along with a detailed exposition of the Rosin-Rammler-Sperling formula. While the RRS formula has been developed primarily for powdered coal and other milled materials, it has been found applicable also to chips produced by turning, milling, and grinding operations, and also to droplet-size distribution in liquid sprays. The RRS formula represents the residuum of a multitude (of particles, dusts, drops) expressed as weights, as ordinate, over the size of the particle (a length dimension) as abscissa. Both coordinates are in log, and log-log scale, respectively. There are recent attempts to use as ordinate not the weight, but the surface area, in view of the main purpose of comminution, i.e., the increase of the surface area. The critical survey of a large number of research studies in this field by the author and by others makes this article particularly useful.

K. J. De Juhasz, Germany

4129. Trofimov, G. I., Noneroding velocity for sandy beds (in Russian), *Gidrotekh. Stroit.* **25**, 1, 50–58, 1956.

Six formulas by Russian authors were checked on 14 sand banks in 4 Russian rivers. Formula by Goncharov (1938) showed closest agreement: $v_x = 3.9\, H^{0.2}\, \varphi^{0.2}(d_m + 0.0014)^{0.3}$ (for metric units); here H is the water depth, d_m average diameter of silt particles, φ is an irregularity factor $\varphi = d_m/d_{90}$, a ratio of the average diameter to the 90% value on a granulometric curve.

S. Kolupaila, USA

4130. Sokolowski, M., Some problems of statics of granular and cohesive bodies (in Polish), *Arch. Mech. stos.* **7**, 1, 3–23, 1955.

Paper is concerned with the theory of statics of granular and cohesive bodies. It presents a concise and clear account of basic assumptions, methods of solving problems, and results obtained by the author, which are of great importance from both the scientific and practical point of view.

Assuming limit equilibrium, author considers in the first part of the paper the two-dimensional problem for generally understood granular bodies, characterized by an angle of internal friction ρ and cohesion value k.

A suitable choice of new variables α and φ, related with the stress components $\sigma_x, \sigma_y, \tau_{xy}$, permits obtaining general methods for solving problems of fundamental importance; e.g., of the stability of foundations, stability of banks, and the problem of the pressure upon retaining walls. Formulas are also given for the more important quantities, which are interesting from the general theory point of view.

The second part of the paper deals with the two-dimensional problems of limit equilibrium in ideal cohesive bodies, deprived of internal friction ($\rho = 0$). By means of considerations arranged in a sequence analogous to that of the first part, author shows how the solution of the specific problems given therein is simplified as a result of initial assumption.

The problem of the half-plane, in the case when some regions are in the elastic state, is broached too. Concluding this part of the paper, author formulates the basic equation of the two-dimensional problem of granular bodies, characterized by a general condition of limit equilibrium.

The third part of the paper is devoted to two-dimensional problems of limit equilibrium of ideal granular bodies, i.e., cohesionless bodies ($k = 0$). Author analyzes these problems in polar coordinates, obtaining the solutions of several basic problems. Due to the special properties of such bodies, solutions may be obtained in a simpler form than would be possible when adopting the general theory. Attention is also drawn to problems in which some regions remain elastic.

Finally, author gives a method of approximate investigation of the problems of statics of granular bodies. It consists in representing the solution as the sum of the solution of the same problem while neglecting the weight of the body ($\gamma = 0$), and the solution while neglecting the cohesion of the body ($k = 0$).

W. Olszak, Poland

4131. Szilágyi, J., Geoelectric method of gaging the velocity of ground water (in Hungarian), *Res. Inst. for Water Resources, Budapest 1954 Report*, 100–108, 1955.

A salt solution is introduced into single drill hole. Location of equipotential points around the well is determined with a potentiometer. Displacement of the center of gravity shows the direction of flow and the velocity of ground water. Results checked by test pumping and Darcy formula were in fairly good agreement. Method is of practical significance. S. Kolupaila, USA

4132. Barenblatt, G. I., Motion of suspended particles in a turbulent flow in a semi-space or in a flat open channel of finite depth (in Russian), *Prikl. Mat. Mekh.* **19**, 1, 61–88, 1955.

4133. Aronofsky, J. S., and Jenkins, R., A simplified analysis of unsteady radial gas flow, *J. Petr. Technol.* **6**, 7, 23–28, July 1954.

A simple means of predicting the flowing well-pressure history in a natural gas reservoir has been developed. The differential equation for unsteady radial flow of gases through porous media was solved by numerical methods using electronic punch card machines. The results obtained from these calculations are presented in a generalized form which is believed to be most convenient for engineering use. These results are compared with those presented in a paper by Bruce, et al.

An effective radius of drainage has been defined which permits the equation of steady-state gas flow to be used easily to predict well pressures in gas reservoir depletion. An equivalent effective radius of drainage is defined for liquid flow systems, and a comparison is made of the gas solution to the liquid solutions.

From authors' summary

Geophysics, Meteorology, Oceanography

(See also Revs. 3857, 4015, 4016, 4021, 4156)

4134. Crease, J., Long waves on a rotating earth in the presence of a semi-infinite barrier, *J. fluid Mech.* **1**, 1, 86–96, May 1956.

A train of long gravity waves of small amplitude approaches a semi-infinite barrier which extends parallel to the wave crests, the whole system being in rotation. Curvature of the earth is neglected. If there is no rotation, there is a shadow region behind the barrier in which the amplitude diminishes rapidly with distance from the edge. It is now shown that for one orientation of the barrier there is an additional wave in the shadow region which travels without attenuation along the barrier. The amplitude just behind the barrier may exceed the amplitude of incident waves. It is suggested that, for the other orientation, a wave travels along the front of the barrier, and that energy may be trapped by a barrier of finite length.

The mathematical formulation leads to an integral equation of Wiener-Hopf type. The solution involves Fresnel integrals of complex argument. F. Ursell, England

4135. **Gutenberg, B., and Richter, C. F., Earthquake magnitude, intensity, energy, and acceleration,** *Bull. seism. Soc. Amer.* **46,** 2, 105-145, Apr. 1956.

Authors present revision of their previous paper on magnitude scale and earthquake energy [title source, **32,** p. 163, 1942]. Discussions are made on factors (e.g., epicentral distances, azimuthal factor, characteristics of seismometers, ground condition of stations, etc.) to which amplitude and period of recorded traces on seismograms are responsive. Magnitude-energy relation is revised as follows:

$$\text{Log } E = 9.4 + 2.14\ M - 0.054\ M^2$$

which is based on equation

$$\text{Log } (A_0/T_0) = -0.76 + 0.91\ M - 0.027\ M^2$$

where A_0 and T_0 denote, respectively, amplitude and period of maximum disturbances at epicenter assumed from traces of some stations. However, complete revision of magnitude scale is not yet given and authors suggest that it will be published in near future, probably being based on A/T rather than amplitudes.

K. Kasahara, Japan

4136. **Heath-Smith, J. R., Turbulence encountered by Comet 1 aircraft,** *Aero. Res. Counc. Lond. curr. Pap.* 248, 26 pp. + 6 figs., 1956.

Acceleration records have been obtained of the turbulence encountered by B.O.A.C. Comet aircraft in 680,000 miles of operational flying on routes connecting London with South Africa and the Far East. It is shown that turbulence decreases with increasing altitude. Of all gusts greater than 10 E.A.S., 91% were encountered below 27,500 ft. Above 27,500 ft, about 75% of gusts greater than 10 fps E.A.S. occur in cumuliform cloud. In flight sectors where conditions favor the formation of cumuliform cloud, gusts may be several times more numerous on average than over widespread routes. From author's summary

Lubrication; Bearings; Wear

4137. **Hagg, A. C., and Sankey, G. O., Some dynamic properties of oil-film journal bearings with reference to the unbalance vibration of rotors,** *J. appl. Mech.* **23,** 2, 302-306, June 1956. See AMR **9,** Rev. 1665.

4138. **Newman, A. D., Bearings for marine geared turbines,** *E. Cst. Instn. Engrs. Ship. Trans.* **72,** part 5, 205-228, Mar. 1956.

The prime requirements of bearings for marine-geared turbines, reliability and efficiency, are considered. It is shown that the correct choice of length/diameter ratio and clearance ratio is essential in order to attain these requirements, and it is indicated that, for the conditions which occur in most marine turbine and gear bearings, relatively short bearings with length/diameter ratio between two-thirds and one-third, and clearance ratios between one and two thousandths of an inch per inch diameter, give the best results.

Bearing and shell materials are discussed. The advantages of high-tin white metal as bearing material are shown, and the relative merits of nonferrous and ferrous shell materials are considered. It is concluded that the best general design is one of thin whitemetal on a steel shell; the shell may be of normal thickness or it may be of thin shell type, and the advantages of this design are considered.

The requirement of utmost reliability in a marine bearing involves if possible the ability for limited operation under a condition of lubricating-oil failure. The shortcomings in this respect of present bearings are mentioned, and an improved design which overcomes them is described. From author's summary

4139. **Barwell, F. T., Grunberg, L., and Scott, D., An investigation of heavy duty lubricating oils by electron microscopy,** *Instn. mech. Engrs. Auto. Div.* no. 5, 153-164, 1954-55.

A number of unused and used heavy duty (H.D.) oils were investigated by the electron microscope. Plain oils containing no additives and an oil containing an anti-wear additive were also investigated. No support was found for the suggestion that unused oils containing H.D. detergent additives are colloidal suspensions, as the micrographs were perfectly clear and no particles were found to be present. Two types of particle were found to be present in used H.D. and plain oils. One type of particle was of the order of 0.05 micron (500A) and tended to arrange itself in chain structures. The other type was very much larger, up to several microns in diameter. The two types of particle may have different origins.

It is concluded that the electron microscope is unlikely to provide a very useful means for the inspection of unused H.D. oils, and the authors express doubts regarding the validity of an explanation of their successful use based solely on their deflocculating or dispersant action. From authors' summary

4140. **Geary, P. J., Knife-edge bearings. A bibliographical survey of instrument parts, no. 2,** B.S.I.R.A. Research Rep. M. 20, 61 pp., 1955.

Marine Engineering Problems

(See also Rev. 4138)

The following 6 papers (Revs. 4141-4146) appeared in the Vedeler Anniversary number of European Shipbldg. *5, 1, pp. 4-29, 1956.*

4141. **Kendrick, S., The analysis of flat plated grillages,** pp. 4-10.

4142. **Muckle, W., The transverse strength of ships,** pp. 11-17.

4143. **Svennerud, A., An approximate method for the calculation of critical compression stresses in cross-stiffeners have a certain degree of fixation,** pp. 18-20.

4144. **Steneroth, E., A successive approximation method for grid calculation,** pp. 21-25.

4145. **Schade, H. A., Stresses in cross-stiffened plate diaphragms,** pp. 26-29.

4146. **Brehme, H., The propeller slip diagram,** p. 29.

Anniversary number is intended to emphasize Vedeler's outstanding achievements within field of shipbuilding industry and his general understanding of ship technical problems. All papers refer to ship strength calculation and are more or less initiated by Vedeler's book "Grillage beams in ships and similar structures" [Oslo, 1945, 187 pp.]. *S. Kendrick* presents an alternative to Vedeler's theory as well as to other "effective breadth" theories, starting from system composed of two orthogonal sets of beams of T-section and a plating in which a plane state of stress prevails.

Treatment is simplified by assumption that plating is assumed to act independently in direction of each set of beams and that, for each set of beams, displacements of plating are considered only parallel to corresponding beam. Solution in terms of intersection reactions is aimed at, and actually produced for, special grillage. For comparison, two alternative grillages, consisting of same T-beams with complementary opposite "effective" flange instead of plating, are considered. They give surprisingly good agreement with author's more exact solution, thus confirming Vedeler's assumptions in this particular case. Compact presentation and numerous misprints make it difficult to check assumptions and follow development of this important paper.

W. Muckle reviews earlier conventional treatment of title subject, pointing to its insufficiency and indicating Vedeler's method of taking account of interaction between transverse and longitudinal stitteners. *A. Svennerud* treats title subject with Vedeler's method, using approximate expression for deflection and energy considerations.

E. Steneroth's paper forms further development of author's earlier work ["On the transverse strength of tankers," *Trans. roy. Inst. Tech. Stockholm*, no. 90, 1955; also, Acta Polyt. no. 166, 1955]. Method is applicable to grids, consisting of several weak beams (frames) and some very heavy beams (webs) crossing the former, as in case for grids appearing in tankers. Intersection reactions of frames and webs are calculated by successive approximations, starting from those reactions necessary to correct deflections produced by external load on frames and webs, considered as independent systems. Advantage of method is that individual steps of calculation always refer to simple beams with loads given from previous step. Convergence of method is studied and found to be very rapid for some simple loading cases.

H. A. Schade considers title structure as orthotropic plate under action of forces in its own plane. Values of fictitious elastic constants are discussed, considering repeating element of structure consisting of plate and two web systems. Solution in terms of Airy's stress function is indicated. Serious misprint is noted in Eq. (11), where one derivative of stress function is missing.

F. K. G. Odqvist, Sweden

4147. Jasper, N. H., Service stresses and motions of the Esso Asheville, a T-2 tanker, including a statistical analysis of experimental data, *David W. Taylor Mod. Basin Rep.* 960, 45 pp., Sept. 1955.

Stresses amidships as well as pitching and heaving accelerations were measured on the T-2 tanker *Esso Asheville* during the vessel's normal commercial operations over the period 24 August 1952 to 3 April 1954. The data were obtained automatically, using a sampling procedure, and are analyzed by statistical methods.

Analysis indicates that the dynamic stress variations associated with the ship's motion in waves will rarely exceed 12,000 psi peak-to-peak. Also, the stress variations due to changes in temperature distribution occasionally are of the same order of magnitude as those due to changes in dead-load distribution. The maximum measured stress variation (free of stress concentration) corresponding to changes in temperature was about 11,000 psi.

The occurrence of wave-induced service stresses and of the ship motions experienced by the *Esso Asheville* may be approximated by a logarithmically normal distribution. Expected extreme wave-induced stresses are predicted by use of the methods of statistics.

From author's summary

4148. Morgan, W. B., An approximate method of obtaining stress in a propeller blade, *David W. Taylor Mod. Basin Rep.* 919, iv + 19 pp., Oct. 1954.

An improved practical method, well adapted to modern thickness forms, enables the designer to obtain minimum thickness for the sections, The blade is still considered not as a cantilever plate, but as a simple beam. The circulation theory is used for the calculation of the thrust and torque distribution on the blade. An accurate method is designed to minimize the work required for calculating the geometric properties of the sections. An example is given.

R. Spronck, Belgium

4149. Verhagen, C. J. D. M., and De Does, J. C., A special stress analyser for use on board ship, *Inter. Shipbldg. Progr.* 3, 21, 285–289, May 1956.

4150. Kumai, T., Some notes on the local vibrations of ships, *Rep. Res. Inst. appl. Mech. Kyushu Univ.* **3,** 12, 171–175, Mar. 1955.

4151. Gerritsma, J., Some notes on the calculation of pitching and heaving in longitudinal waves, *Inter. Shipbldg. Progr.* **3,** 21, 255–264, May 1956.

This is perhaps the most thorough evaluation of the magnitude of exciting forces and moments in heave and pitch, based on the classical Froude-Krylov hypothesis. It is shown that by applying the Smith effect correction, and that due to relative motion, good agreement with measurements have been obtained. Although this result may be incidental, it indicates that, for the purpose of general orientation, the classical approach still deserves attention. For more detailed investigations, boundary conditions on the body must be fulfilled.

G. P. Weinblum, Germany

Biomechanics

(See also Rev. 3862)

Books Received for Review

BOSWORTH, R. C. L., Transport processes in applied chemistry, New York, John Wiley & Sons, Inc., 1956, x + 387 pp. $12.

FINCH, V. C., Jet propulsion. Turbojets, revised ed., Palo Alto, Calif., The National Press, 1955, xv + 327 pp. $5.50.

FINCH, V. C., Jet propulsion. Turboprops, Palo Alto, Calif., The National Press, 1950, xvi + 256 pp.

JOHNSON, C. L., Analog computer techniques, New York, McGraw-Hill Book Co., Inc., 1956, xi + 264 pp. $6.

Proceedings of the second Midwestern conference on solid mechanics, Purdue Univ. Sept. 8, 9, 1955, Lafayette, Ind., Purdue University, Purdue Engng. Sta. Res. Bull. 129, 249 pp. $5.

RICHARDS, R. K., Arithmetic operations in digital computers, 4th ed., D. Van Nostrand Company, Inc., 1956, iv + 397 pp. $8.

SUTTON, G. P., Rocket propulsion elements, 2nd ed., New York, John Wiley & Sons, Inc., 1956, x + 483 pp. $10.25.

Symposium Sobre Applicaciones de la Plasticidad en el Calculo de las Estructuras de Hormigon Armado, 4 papers, Organizado por la Comisión Téchnica de Estructuras y Analisis Experimental de Tensiones de la Sociedad Argentina de Ensayos de Materiales, Buenos Aires, Librería "El Atenio" Editorial, 1955, 122 pp.

WILSON, E. D., and RIES, H. C., Principles of chemical engineering thermodynamics (McGraw-Hill Series in Chemical Engineering), New York, McGraw-Hill Book Co., Inc., 1956, ix + 376 pp. $7.50.

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